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THE
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ARTEMAS MARTIN, PH.D.

WASHINGTON, D. C.

A NEW, COPIOUS, AND COMPLETE SYSTEM OF ARITHMETIC,

FOR THE USE OF

SCHOOLS AND COUNTING-HOUSES,

IN THE UNITED STATES OF AMERICA;

OR

ARITHMETICAL INSTRUCTOR;

CONTAINING A GENERAL COURSE OF

MERCANTILE EXAMPLES,

TOGETHER WITH

A VARIETY OF SPECULATIVE MATTERS,

WELL ADAPTED TO AMUSE, AND AT THE SAME TIME IMPROVE THE UNDERSTANDING.

BY JAMES MAGINNESS, Esq.

HARRISBURG:
PRINTED FOR THE AUTHOR,
BY WILLIAM GREER.
1821.

QA
101
M19

EASTERN DISTRICT OF PENNSYLVANIA, TO WIT:

BE IT REMEMBERED, That on the thirtieth day of July, in the forty-sixth year of the Independence of the United States of America, A. D. 1821, JAMES MAGINNESS, of the said district, hath deposited in this office, the title of a book, the right whereof he claims as author in the words following, to wit:

"A new, copious, and complete system of Arithmetic, for the use of schools and counting-houses, in the United States of America; or Arithmetical Instructor; containing a general course of mercantile examples, together with a variety of speculative matters, well adapted to amuse, and at the same time improve the understanding. By James Maginness, Esq."

In conformity to the act of the Congress of the United States, intituled, "An act for the encouragement of learning, by securing the copies of Maps, Charts, and Books, to the authors and proprietors of such copies, during the times therein mentioned;" and also to the act, entitled, "An act supplementary to an act, entitled, "An act for the encouragement of learning, by securing the copies of Maps, Charts, and Books, to the authors and proprietors of such copies during the times therein mentioned," and extending the benefits thereof to the arts of designing, engraving, and etching historical and other prints."

D. CALDWELL, clerk of the eastern
district of Pennsylvania.

Hist of science
Amer. Univ.
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22.50 2.50
1.25 4.25

PREFACE.

THE author of the *Arithmetical Instructor* begs leave to make a few remarks to the public, upon his work. First. He did not undertake the work, because there was an apparent dearth of Arithmetical books in being; no, there appears to be no scarcity of such works; and, indeed, it may by some be considered vanity in him, to suppose, that by adding another Arithmetical work to the already full swelled list, he has contributed much to the advantage or benefit of society; or added any thing material, which can be called new. A perusal of the work, will, he hopes, remove those prejudices, if any such exist. Secondly. He did not undertake the work for the purpose of underrating and browbeating such writers of Arithmetic, as have chosen to discuss and arrange their books differently from what he might fancy to be a better plan. He undertook it, principally, for the purpose of rendering the subject, if possible,

35036

more perspicuous and demonstrative, than any of those systems of Arithmetic in common use. In this object, how far he has succeeded, will be for a generous and impartial public to determine. His plan throughout, was to preserve originality as much as possible; when this failed, he made it a point to select from the best authors which he could lay his hands upon, such rules and examples as he considered congenial with the general plan of the work. The chapter on Exchange, is chiefly selected from authorities which have been previously published; some other articles have also been selected in like manner. He has taken care particularly to demonstrate and illustrate most of the rules which appeared to be obscure or ambiguous; and used his utmost exertions to render it as perfect as the nature of such works will admit of, both in point of giving the true answers to the respective questions, and in doing many of them in a proper mathematical style. He would observe, that in two or three instances, articles have been rather prematurely inserted, viz. as in pages 166, 175, &c. In the course of the work, a number of Algebraic operations have been here and there introduced; but he hopes, that as every thing is, in general, performed Arithmetically, he will be excused for using them. Logarithms have been used for the solution of a few cases in Geometrical Progression and Annuities; but it must be recollected, that in these cases, Logarithms are indispensable. If any error, either in principle or calculation, should escape the author's observation, he hopes that his readers will remember, that such works are almost inseparable from minute errors; and that if any be discovered, he will thankfully receive information thereof, from such as will point them out in a respectful manner. As all the questions in the Single Rule of Three Direct, Inverse, and Double Rule of Three, are stated, and the remaining most difficult cases resolved, the work will supercede the necessity of a key. Any teacher of moderate capacity can, from what is shewn, easily conduct a student through it from beginning to end.

Upon a careful examination of the Arithmetical Instructor, it will be found that a number of new and highly useful rules have been introduced, others improved and explained; which, taking the whole together, for originality and usefulness, the author is not afraid to compare with any book of Arithmetic, which he has yet seen published; and confidently submits it to an impartial public.

A list of fulsome recommendations could have been easily procured; but well knowing, that if a book has not a decent share of merit, it matters little who recommends it, it will not long claim the patronage of an enlightened public.

It may, perhaps be objected to by some, that he has resolved too many of the problems in some of the easy rules; this he did for the purpose of shewing neat and concise methods of solution, in practical matters which immediately relate to business. People in the interior of the country may, perhaps, object to the length of the chapter on Exchange; but upon a little reflection, they will find, that the historical instruction which it contains, will amply compensate for its room; to merchants on the seaboard, it will be indispensable. He has shewn, by a single example in each case, how to resolve all the problems belonging to Arithmetical and Geometrical Progression; he has also shewn how to raise the theorems, and perform one example, in each case of Simple Interest; the present worth of Annuities or Pensions; Annuities, Leases, &c. taken in reversion; Rebate and Discount; Compound Interest; present worth of Annuities, Pensions, &c. computed in the same way; and Purchasing Freehold Estates in reversion. It will not, he trusts, be thought out of place, that he has introduced an easy method of making the Logarithms of natural numbers, and also the Arithmetic of Logarithms. This subject might, by some, be thought more properly to belong to a system of Mathematics; but as books of Arithmetic more frequently fall into the hands of youth, than those of Mathematics, he did it through a design of exciting students of Arithmetic to higher studies, viz. to that of Mathematics, as soon as they shall have passed through the rules of Arithmetic.

He professes to be the public's humble servant.

J. M.

Harrisburg, Sept. 20, 1821.

☞ NOTE. The author intended to have annexed a list of subscribers' names; but not receiving many of them time enough, nor some of them at all, he concluded that it were better not to insert any.

ADVICE TO STUDENTS.

The author having taught a number of years, found it necessary to impress students of Arithmetic and Mathematics, as they progressed, with the necessity of committing all the useful rules and definitions to memory; and also, to endeavour to comprehend the meaning, use, and application of the examples arising out of those rules and definitions. This enables them to command the necessary helps, at a moment's warning, without book or any thing else. It looks awkward in a man who pretends to be a scholar, to run to his books for every thing he may want in the course of business. It is true, that where the use of Mathematical tables is required in the investigation of Geometrical and other problems, books are to be adverted to. It is no where expected, that tables of logarithms, tables of latitudes and departures, and many others, can be committed to memory; but wherever it is practicable, it should be done. Useful rules and theorems should be deeply impressed upon the mind. Every useful definition should be clearly understood by the student, as he progresses; then, what would otherwise be mysterious, will appear as clear as the sun at noon day.

EXPLANATION OF CHARACTERS.

* † ‡ ‖	Where any of these marks are found in the body of the *Arithmetical Instructor*, look to the bottom of the page for their corresponding marks, and you will find a further explanation of what is spoken of above.
$+$	When this sign is found between two numbers, or quantities, it shews that they are added, and is therefore the sign of addition, called *plus*.
$-$	When this sign is found between two numbers, or quantities, it shews that they are subtracted, and is therefore the sign of subtraction, called *minus*.
$\times$	When this sign is found between two numbers, or quantities, it shews that they are multiplied, and is therefore the sign of *multiplication*.
$=$	This is the sign of equality, and reads *equal*, and shews that where it is placed between two numbers, or quantities, that these numbers and quantities are *equal*.
$\div$	This is the sign of *division*; and shews, when placed between two numbers, or quantities, that the first is divided by the last.
$\frac{16}{4}$ or $\frac{a}{b}$	When numbers, or quantities, are placed fraction-wise, as those in the margin, it shews that the upper number, or quantity, is divided by the lower one.
3)12(4	This shews, that 12 divided by 3 is equal to 4.
$2 : 4 :: 8 : 16$ $a : b :: c : d$	When dots are placed between numbers, or quantities, as in the margin, they are called signs of proportion, and read, viz. as 2 is to 4, so is 8 to 16, or, as *a* is to *b*, so is *c* to *d*.
$\frac{bc}{a}=d$	This expression is called an equation, and means that *b*, multiplied by c, and divided by *a*, gives a quotient equal to *d*.
$ptr+p=a$	This expression is also an equation, which, in Simple Interest, means that the principal, time, and rate per cent, plus the principal, is equal to the amount.

$pr^t=a$	This equation, in Compound Interest, means that the principal multiplied by the ratio, raised to the power of the time, is equal to the amount.
$2\times1=2^1$, $a\times1=a^1$	Shews that 2 multiplied by 1, and a mutiplied by 1, give their first powers.
$2\times2=2^2$, $a\times a=a^2$	Shews that 2 multiplied by 2, and a multiplied by a, give their second powers, or squares.
$2\times2\times2=2^3$ $a\times a\times a=a^3$	Shews the third powers of 2 and a, or their cubes.

And, in general, 2^n and a^n shew any powers of 2 and a. $(1+7)^2$ and $(a+b)^2$, or $(2+7)^n$ and $(a+b)^n$, shew that the first pair, inclosed in parenthesis, are squared, and the second pair raised to any power.

$\surd 2$ or $\sqrt{2}$ $\surd a$ or $\sqrt{a}$	When such characters are placed before numbers, or quantities, it shews that their square roots are extracted, and is therefore the sign of *evolution*.
$(2+7)^{\frac{1}{2}}$ $(a+b)^{\frac{1}{2}}$	These expressions shew that the square roots of $2+7$, and $a+b$, are extracted.
$\sqrt[3]{}2$ or $\sqrt[3]{2}$ $\sqrt[3]{}a$ or $\sqrt[3]{a}$	These expressions shew that the cube roots of 2 and a are extracted.

And generally,

$\sqrt[n]{}2$ or $\sqrt[n]{2}$ $\sqrt[n]{}a$ or $\sqrt[n]{a}$	When the exponent of the root is expressed by n, it means that the nth roots of 2 and a are extracted; that is, any roots, &c.
$\dfrac{\cancel{2}\times\cancel{3}\times\cancel{4}\times\cancel{5}\times7}{\underset{\cancel{2}}{\cancel{6}}\times\cancel{4}\times9\times\cancel{5}}=\frac{7}{9}$	These figures, with the dash drawn across them, shew that they are cancelled; that is, that equals are stricken from both dividend and divisor.
$<$	This character, placed between two numbers, or quantities, shews that the one on the left is less than that on the right.
$>$	This, the reverse of the above.
$\pm$	Where this ambiguous sign is placed between two quantities, it signifies that sometimes both are to be added, and sometimes the one subtracted from the other.
♭	This character, in music, denotes a flat.
#	This, a sharp.

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ERRATA.

Page.

16 lines 17 and 24 for *Georgian*, read Georgium.

38 18 same error.

71 14 operation begins erroneous and so continues throughout.

86 13 and 14 for *lb.* at the second 10, read *Hh.*

101 18 for the numerators 350, 450, read 360, 440.

112 last line, for $\frac{34}{83}+\frac{49}{83}$, read $\frac{34}{83}\times\frac{49}{83}$.

141 16 for $\frac{7}{5}$, read $\frac{7}{500}$.

163 23 strike out *made by them.* End of same line, beginning of line 24, strike out *on every* $100.

164 16 for *factors*, read factor.

185 13 for *is*, read are.

190 10 note, for $\sqrt[3]{16x}$, read $\sqrt{16x}$.

270 3 after 17½, read +6¼.

279 5 for $2+\sqrt{1.25}+$ &c. read $2+\sqrt{1.25}\times$ &c.

289 4 for $\frac{\overline{z+a}+\overline{z-a}}{2s-z-a}$, read $\frac{\overline{z+a}\times\overline{z-a}}{2s-z-a}$

304 for. 11 $\overline{\frac{18}{2}\Big]}\Big|\frac{1}{3-1}\sqrt{9}=3$, read $\overline{\frac{18}{2}\Big]}\Big|\frac{1}{3-1}=\sqrt{9}=3$

306 form. 19 after 2×26—, place 18— before the radical sign.

323 lines 19 and 27, for 2, read 3, and for 31, read 21. 28, read *d*8, &c.

345 6 for 9)9.289, &c. read 9)0.289, &c.

346 10 for 4, read 5. Line 12, for 3, read 4.

ARITHMETICAL

INSTRUCTOR.

ARITHMETIC.*

1. The object of Arithmetic is, to represent numbers in a concise and distinct manner by characters, and to simplify, by means of such characters, the methods of calculation.

2. Arithmetic may be called an art, when it merely teaches the methods of performing the practical operations to which numbers are subjected.

---

* According to Ash's English Dictionary, *Arithmetic* is derived from the Greek, (*Arithmos*, number, and *Metreo*, to measure,) the science of numbers, the art of computation.

Samuel Webber, in his Mathematics, says, that number is the abstract ratio of one quantity to another of the same kind, taken for unity; and that Arithmetic, as a science, is the theory of numbers, and as an art, the method of practical operations in numbers.

The Abbot Demolieres says, that quantity *discrete* is called number, of which the most simple is unity, or one; and that science which treats of number or quantity *discrete* is called Arithmetic.

3. It may be called a science, when it demonstrates the various properties and relations of numbers.

4. The nature of Arithmetical operations is, to find from numbers given, others which are required.

5. There are five principal rules in Arithmetic, viz. Notation, Addition, Subtraction, Multiplication, and Division.

6. To these belong Numeration, which is the method of expressing any number in words.

NOTATION.

1. Notation of numbers is, the proper method of writing them.

2. Numbers are usually expressed by certain Arabic characters, called figures, and the Roman letters.

3. The Arabic figures are thus expressed, viz. 1, one; 2, two; 3, three; 4, four; 5, five; 6, six; 7, seven; 8, eight; 9, nine; 0, cypher.

4. These ten characters, being properly disposed, will conveniently express any number, however great.

5. The Roman letters will, also, express any number, without limitation; but are chiefly used to mark dates, and the chapters and sections of books.

6. The following letters are used to express numbers, viz.

I.	V.	X.	L.	C.	D.	M.
1.	5.	10.	50.	100.	500.	1000.

NUMERATION.*

Quadrillions.	Hundreds of thousands of trillions.	Tens of thousands of trillions.	Thousands of trillions.	Hundreds of trillions.	Tens of trillions.	Trillions.	Hundreds of thousands of billions.	Tens of thousands of billions.	Thousands of billions.	Hundreds of billions.	Tens of billions.	Billions.	Hundreds of thousands of millions.	Tens of thousands of millions.	Thousands of millions.	Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	
1	1	2	3	4	5	6	7	8	9	9	8	7	6	5	4	3	2	1	2	3	4	3	2	1	
5.	4.						3.						2.						1.						Period.

Ten is written thus,	10
Nineteen,	19
Thirty-nine,	39
Two hundred and five are written thus,	205
Three hundred and sixty,	360
Ten thousand five hundred and thirty,	10530
Six millions two hundred thousand,	6200000
Three hundred and seventy millions four hundred and ninety-two thousand,	370492000

It is required to write down in proper figures the numbers expressed by the following words, viz.

Twenty-nine.

One hundred and nineteen.

---

* Any number of figures whatever, may be accurately numerated, by dividing them into periods of six figures each, as in the above scheme, repeating millions of millions, for billions, millions of millions of millions for trillions, &c. as often as the figures 2, 3, 4, &c. placed underneath, in the above scheme, designates.

Three thousand and nine.

Ten thousand and nineteen.

Six hundred thousand three hundred.

Nine hundred millions, three hundred thousand, three hundred and three.

Seventy-five billions.

One hundred quadrillions.

It is required to write down in proper words the following numbers, viz.

The diameters of our Moon,	Are expressed in miles by the numbers opposite to each, viz.	2180.
Mercury,		2600.
Venus,		7906.
Earth,		7970.
Mars,		4444.
Jupiter,		81000.
Saturn,		67000.
Georgian Sidus,		35000.

The diameter of the sun,	Are expressed in miles by the numbers opposite to each, viz.	763000.
The mean distance of the Moon from the Earth,		240000.
The distances of Mars,		125000000.
Jupiter,		426000000.
Saturn,		780000000.
and Georgian Sidus, from the Sun.		1565000000.

Archimedes said, "give me a place to stand, and I shall remove the Earth." The probable weight which Archimedes would have had to raise, is expressed by the following number of pounds, viz.

400 000 000 000 000 000 000 000. How many?

NOTATION

OF THE ROMAN LETTERS.

Units.		Tens.	
I	1	X	10
II	2	XX	20
III	3	XXX	30
IIII or IV	4	XXXX or XL	40
V	5	L	50
VI	6	LX	60
VII	7	LXX	70
VIII	8	LXXX	80
VIIII or IX	9	LXXXX or XC	90

Hundreds.		Thousands.	
C	100	CIƆ or M	1000
CC	200	MM or $\overline{\text{II}}$	2000
CCC	300	MMM or $\overline{\text{III}}$	3000
CCCC or CD	400	MMMM or $\overline{\text{IV}}$	4000
D or IƆ	500	IƆƆ or $\overline{\text{V}}$	5000
DC or IƆC	600	IƆƆM or $\overline{\text{VI}}$	6000
DCC or IƆCC	700	IƆƆMM or $\overline{\text{VII}}$	7000
DCCC or IƆCCC	800	IƆƆMMM or $\overline{\text{VIII}}$	8000
DCCCC or IƆCCCC	900	IƆƆMMMM or $\overline{\text{IX}}$	9000

CCIƆƆ or $\overline{\text{X}}$	10000
CCIƆƆCCIƆƆ or $\overline{\text{XX}}$	20000
MDCCCXX	1820

SIMPLE ADDITION.

Simple Addition is the art of summing two or more *pure** whole numbers, and finding a new one, which is called the *sum* or *aggregate* of those numbers.

According to our present Notation, the number 10 is fixed for the universal *ratio*† of *pure* whole numbers, whence arises the following rule:

1. Prepare the numbers to be added, by placing units‡ under units, tens under tens, &c. in perpendicular columns from right to left.

2. Begin to add at the bottom of the right hand column, count the figures up to the top; if the sum be below the *ratio* 10, set it underneath, and proceed to the next; but if such sum be 10, or upwards, a cypher, or the excess of the ten or tens, must be set down, and one added to the next column for each

---

* Pure or abstract numbers are such as are considered without regard to kinds or denominations.

† Ratio properly belongs to the doctrine of magnitudes; it is the mutual relation of magnitudes of the same kind to each other in respect to quantity. Def. 3. Book 5. Euclid.

‡ By Mathematical Correspondent, page 117, vol. 1, a unit is the smallest part of a number, and numbers begin their increase at the smallest place, viz. that of unity; and every number, small and great, contains a definite plurality of units, therefore every number is a definite plurality of units.

Note. When we add pure whole numbers, it is well enough to observe, when the first column is counted, that if the sum exceeds 10, the right hand figure is only to be set down, and the left added to the next column. This caution must be observed throughout, to the last or left hand rank, whose sum must be all set down.

ten in the preceding one; thus proceed to the last column, which add and set down the whole, and the thing is done.

EXAMPLES.*

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9

236562	372145	140484	748045
100001	403400	109101	101140
131225	401332	920401	110712
922210	401102	100012	900001

---

* Simple Addition may be proved several ways; one of which is, to count downwards, in the manner that you first counted upwards; if both results agree, you may conclude the work is right. A second method of proof arises from a property of the numbers 9 and 3, which only belong to them as an effect of our present notation, viz. that any number divided by 9 or 3, or the sum of its digits divided by 9 or 3, will leave equal remainders. Demonstration. Let $r=10$, and a, b, c, &c. the digits of any number in our notation, the number itself will be ar^2+br+c from which taking the sum of its digits, $a+b+c$ there remains $a\times\overline{r^2-1}+b\times\overline{r-1}=(a\times\overline{r+1}+b)\times\overline{r-1}$ $=(a\times\overline{r+1}+b)\times9$: Since, therefore, the difference between any number and the sum of its digits is divisible by 9, or 3, it evidently follows, that the number and sum of its digits, divided by 9, or 3, leave either none or equal remainders. Hence, from the above demonstration, the following method of proving Simple Addition is derived,

74674673	47404748	47594674
25325327	52595252	52406326
19467814	14674814	74674239
80532186	85325186	26325761
19416483	14848148	27467468
19467483	14748946	98735874

SIMPLE SUBTRACTION.

1. Simple Subtraction is the method of finding the difference between any two *pure* whole numbers, without regard to kinds or denominations.

2. In order to prepare numbers for subtraction, the greater must be placed above the lesser, units under units, tens under tens, &c. as in Addition.

---

viz. Add the figures in each row from left to right, and find how many 9s their sums contain, placing the excess of 9s, in each row, to its right in a perpendicular column, and find their sum. If the excess of 9s, in the sum of the excesses, be equal to the excess of 9s in the sum total, it is right.

EXAMPLE.

3469 . . 4 first excess of 9s.
4234 . . 4
5673 . . 3

Sum total = 13376 . . 11 = sum of excesses.

2 2

The excess of 9s in the sum total, and in the sum of excesses, being each equal 2, it is right.

3. The greater is called the *minuend*, the lesser *subtrahend*, and the number which results from the operation of subtraction is called the *difference*.

RULE.

Begin at unit's place; if the lower figure be less than the one above it, take the difference and set it underneath; proceed to the next in the same manner, and so on to the last, still noting the difference as before: but if the lower figure should sometimes happen to be greater than that above it, the ratio 10 must be first added to such upper figure, and then the difference taken and noted as before, taking care to add one to the next lower figure whenever such circumstance occurs, to the work is done.

EXAMPLES.*

746739467
413213321

94674321
14064110

97463420
14032120

400474014064
127314184372

437004640404
140734041840

17480404674
8501515785

904904048046
174648374177

---

* Subtraction is proved by adding the difference to the lesser, which, if equal to the greater, it is right.

SIMPLE MULTIPLICATION.

1. Simple Multiplication is a concise way of performing extensive additions of simple numbers.*

2. The parts of Multiplication are, 1. The *multiplicand*, or part to be multiplied. 2. The *multiplier*, or part to be multiplied by. 3. The *product*, or *rectangle*.

---

* Mathematical Correspondent, § 4. The denomination of a part of any number, is the number which expresses how often that number contains the part. 4 is named the third part of 12, from the denominator 3, which expresses how often 12 contains its part 4. The denominator of a part is a pure number, and can never be of any particular kind whatever. For, although 4 men be a part of 12 men, yet the number 3 is a *pure* number, independent of every particular kind or denomination. Hence, in the rule of multiplication, the product is a multiple of the multiplicand, (that is, it contains it a certain number of times exactly) consequently the multiplicand is a part of the product, and the multiplier is the denominator of that part. The product is therefore always of the same kind with the multiplicand, and the multiplier is always a *pure* number. Hence, a number of one kind cannot be multiplied by a number either of the same, or of a different kind: 3*l.* cannot be multiplied by 5*l.*, 7*ft.* cannot be multiplied by 4*ft.*; so that the multiplier is always to be considered as a *pure* abstract number, of no particular kind or denomination. The multiplicand and product may be always of the same particular kind or denomination, and the multiplier a *pure* abstract number without kind or denomination.

MULTIPLICATION TABLE,

OR TABLE OF PRODUCTS.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

RULE.

1. Multiply the first figure of the multiplicand by the first figure of the multiplier; if the product be less than 10, set it down in units' place; but if more than 10, or 10s, set down the excess only, and carry or add one to the next left hand product, for each 10, as in addition; and so proceed to the work is done.

2. Begin the operations with multipliers, running from 2 to 12; use them at once, and call this part

CASE 1.

EXAMPLES.

Multiplicand	3420421423		438422143
Multiplier	2		3
Product			
	346704075	123467463	987464301
	4	5	6
	67846434601	3704210474	98343274
	7	8	9
	4746320104	746746749	494674649
	10	11	12

CASE 2.

When the multiplier consists of more figures than one, (but above 20, as 23, 29, 31, &c.) multiply each figure of the multiplicand, by the right hand figure of the multiplier, and then by the second, third, &c. placing the unit's figure of each product under its multiplier. This done, and the products added together, will be the whole rectangle, or answer.

EXAMPLES.

497437463		94674643	945746423
23		79	87
1492312389	$$\begin{array}{ccc} & 1 & \\ 2 & \mathsf{X} & 5^{*} \\ & 1 & \end{array}$$		
994874926			
11441061649	Answer.		

47469321	87464214	9464049
9137	9876	4789

---

* Multiplication may be proved by the cross, as follows, viz. Make a cross like an X, cast the 9s out of the multiplicand first, place the excess on the left; next cast the 9s out of the multiplier, and place the excess on the right; multiply the excesses together, cast the 9s out of their product, and place the excess on the top of the cross; lastly, cast the 9s out of the rectangle or product; if this last excess be equal to that at top, the product is right.

Note. If the excess of the multiplicand be 0, the excess of the product will also be 0; and so of the multiplier. This rule is easily demonstrated, but is here omitted, with this remark, that by going over each operation twice with proper attention, there will be little use for the rule of proof.

CASE 3.

When the multiplier is a *composite** number, multiply the figures of the multiplicand first, by one of the component parts of such multiplier, and the product thence arising by the other, and the thing is done.

EXAMPLES.

Mul. 3456789 by 25 = 5 × 5

$$\begin{array}{rr} & 3456789 \\ & 5 \\ \hline & 17283945 \\ & 5 \\ \hline \text{Pr.} & 86419725 \\ \hline \end{array}$$

7497487 by 27 = 3 × 9

$$\begin{array}{r} 7497487 \\ 3 \\ \hline \\ \hline \\ \hline \end{array}$$

7467456 × 49 = 7 × 7

497643 × 99 = 9 × 11

CASE 4.

When there are cyphers on the right hand of either of the factors, or both, place the significant figures under each other; multiply them together only; to this product annex the neglected cyphers of both factors, and you obtain the product required.

---

* A composite number is, that which is produced by multiplying together any two numbers whatever, and these numbers are called the component parts of such composite number. A composite number is also a multiple of all its component parts; that is, it contains its component parts each a certain number of times exactly. 12 is a composite or multiple number, and its component parts are 1, 2, 3, 4, 6 and 12; for 12÷1=12, $\frac{12}{2}=6$, $\frac{12}{3}=4$, $\frac{12}{4}=3$, $\frac{12}{6}=2$, and $\frac{12}{12}=1$. Therefore it follows, that 1×12=2×6=3×4=12. Whence it is easy to see the reason of the rule.

EXAMPLES.

34674047
7900

312066423
242718329

273924971300

7467309400 × 93000
93000

CASE 5.

When there are cyphers between the significant figures of the multiplier, pass them, and multiply by the significant figures only, setting the first figure of each product directly under its multiplier. Proceed as in case 2d, and you will obtain the product required.

EXAMPLES.

74674324 × 30047
30047

522720268
298697296
224022972

2243739413228

94004369 × 30094
30094

CASE 6.

When any number is to be multiplied by 10, 100, 1000, &c. it is only necessary to annex to such number as many cyphers as belong to the multiplier, and the thing is done.

EXAMPLES.

Multiply	94674876 by 10	94674804 × 100
	10	100
	946748760	

CASE 7.*

To multiply by any number of 9s at once, place to the right of the multiplicand as many cyphers, or dots, as there are 9s in the multiplier; subtract the given multiplicand from this new one, and the thing is done.

EXAMPLES.

Multiply	74673 by 99	7946748 × 999
Operation	74673 . .	
	74673	
Product	7392627	

---

[*] Any sum multiplied by 9, will produce nine-tenths of the same sum multipled by 10; therefore, if a cypher be annexed to the multiplicand, it increases it ten fold; if multiplied by 9, it increases it nine fold; the difference of these products is evidently equal to once the given multiplicand; whence the reason for subtracting it as above.

CASE 8.

When the multiplier is any number between 12 and 20, as 13, 14, &c. the operation can be performed at once, thus: Multiply by the right hand figure of the multiplier only, taking care to add to each product, that figure of the multiplicand which stands to the right of the one multiplied; carrying for every ten as in other cases.

EXAMPLES.

1.		
3467463 × 13*	9467401 × 17	9847404 × 19
45077019		

In example 1. Say 3 times 3 are 9, which put down; again, say 3 times 6 are 18, and 3, its right hand figure, make 21, set down 1, and carry 2; next, say 3 times 4 are 12 and 2 make 14 and 6, 20, 0 and carry 2. So proceed to the last; what you carry then, add to the last figure, and the thing is done.

I would here observe, that there are rules for multiplying by any number of figures in one line; but when the multipliers consist of such numbers as the following, viz. 23, 41, 111, 119, 108, 3749, the operations become too inconvenient for practice; I therefore omit them.

---

* $3467463 \times 13 = 45077019. = \overline{3 \times 3} + \overline{6 \times 3} + 3 + \overline{4 \times 3} + 2 + 6 + \overline{7 \times 3} + 2 + 4 + \overline{6 \times 3} + 2 + 7 + \overline{4 \times 3} + 2 + 6 + \overline{3 \times 3} + 2 + 4 + 1 + 3 = 45077019.$

SIMPLE DIVISION.

1. Simple Division is the method of measuring one *pure* number by another, or a concise way of performing extensive subtractions.

2. Division consists of four parts: 1. The *dividend*, or number to be divided. 2. The *divisor*, or number to divide by. 3. The *quotient*, or answer. 4. The *remainder*, which is either 0, or some number less than the divisor.

RULE.

1. When the divisor is any number under 12, try how often it is contained in one, two, or three of the first, or left-hand figures of the dividend; draw a line and place the quotient figure below.

2. If one, two, &c. remain after division, it is ten, or tens, to be added to the next figure of the dividend. Try the divisor into this sum as before; place the second quotient figure to the right of the first, and so proceed to the last, and the thing is done.

3. Division is proved by multiplying the quotient by the divisor, and adding to the product the remainder (if any thing): this being always equal to the dividend, if the quotient be right.

EXAMPLES.

Dividend.
Divisor. 2)482464 3)4634367 4)467423
Quotient. 241232

5)647434 6)9253675 7)8595278 8)763595

9)793675 10)853136 11)795683 12)8653758

CASE 2.

1. When the divisor exceeds 12, as 23, 29, &c. try how often such divisor will go into the first two, three, &c. figures of the dividend, as in case first.

2. Place the quotient figure to the right of the dividend, multiply the divisor by it, place the product under that part of the dividend which you measured, and subtract.

3. To the remainder, bring down that figure of the dividend which stands next on the right; try the divisor into this, place the result in the quotient, multiply and subtract as before, until all the figures of the dividend are brought down and measured, and the thing is done.

EXAMPLES.

```
23)47464674(2063681
   46······        23
   ——        ———————
    146      6191043
    138     4127362
    ———           11 remainder.
      84     ———————
      69    47464674 Proof, because this
      ——               product is = to the
      156              dividend.
      138
      ———
       187
       184
       ———
         34
         23
         ——
         11 remainder.
```

Operation. The divisor, 23, goes twice into the two first figures of the dividend; I then place 2 in the quotient. Next, I multiply the divisor, 23, by 2, which I place below 47, (the two first figures of the dividend,) I now subtract, and to the difference, 14, I bring down 4, the next right hand figure of the dividend; so I proceed to the last, and find that 23 is contained in 47464674, 2063681 times, and 11 over.

29)32678657(
31)37926578(
39)78652658(
47)39267832(
83)11965876(

327)372675369(
9564)115036750(
19267)965276938(
109467)398567856(
1928656)3232867856(

CASE 3.

1. When the divisor has any number of cyphers placed to the right of the significant figures, reject them when you divide, as well as a like number of figures on the right of the dividend.

2. When the operation of division is finished, annex those figures of the dividend to the remainder, which was rejected, on account of the cyphers of the divisor not being used, and the amount will be the proper remainder which would have resulted had the whole divisor been used.

EXAMPLES.

```
734,00)946474,64(1289
       734...      734
       ----       ----
       2124       5156
       1468      3867
       ----     9023
        6567       34864 remainder.
        5872    ---------
        ----    94647464 proof.
         6954
         6606
         ----
          34864
```

In the above operation, I only used the significant figures, 734, of the divisor, and for the cyphers omitted, I rejected 64, (as many figures to the right of the dividend;) when the operation was finished, 348 remained, to which I annexed the rejected 64, and the amount, 34864, was the whole remainder.

3509000)958362608(
10090000)32678568009(
69500000)7956385637(
55000000)3578572638(

In dividing by 10, 100, 1000, 10000, it is only necessary to strike from the right of the dividend as many figures, as the divisor contains cyphers, and the thing is done, the stricken off figures being the remainder.

EXAMPLES.

Divide 3786539 by 100

100)37865,39(now it is observable that 37865 is the quotient, and 39 the remainder.

1000)386,935

386 quotient.
and 935 remainder.

CASE 4.

1. When the divisor is a composite number, divide by the component parts of such number; first by the one, and the quotient by the other.

2. If a remainder be left at each division, multiply the last remainder by the first divisor, add to this the first remainder, and the sum will be the proper remainder in full.

EXAMPLES.

Divide 37464304 by 25.

Operation 5 × 5 being equal to 25
Frst 5)37464306

Second 5)7492861—1 first remainder.

Quotient 1498572—1 second remainder.

5 × 1 + 1 = 6 whole remainder.

30 = 5 × 6 divisors for 30)206875670(
36 = 6 × 6 do. for 36)967267862(
42 = 7 × 6 do. for 42)926786356(
48 = 6 × 8 do. for 48)359862386(
110 = 10 × 11 do. for 110)936786235(
144 = 12 × 12 do. for 144)516784293(

When the divisor and dividend are even numbers, it often happens that they may be diminished equally by some common divisor, in such a manner as to abridge the work very much.

EXAMPLE.

Divide 72480000 by 1200.

Operation 12,00)724800,00

60400 quotient required.

Note. We are not limited to any particular divisors in the above operation.

CASE 5.

1. By case fifth, contracted Multiplication may be performed as follows, viz.

2. When the multiplier is any even part of 100, 1000, &c. increase the multiplicand by annexing as many cyphers to its right, as there are places in the multiplier.

3. Divide the multiplier thus augmented by the denominator of whatever part the multiplier is of 100, &c. &c. and the thing is done.

EXAMPLES.

Multiply 783467 by $33\frac{1}{3}$*

Operation. $33\frac{1}{3}$ is equal to
$\frac{1}{3}$ of 100, for $33\frac{1}{3}=\frac{100}{3}$
& per rule 3)78346700

Product $26115566\frac{2}{3}$

894632 by 250

250 is = to $\frac{1}{4}$ of 1000
therefore,
4)894632000

223658000 Ans'r.

Multiply 32862583 by 500

9258206 × 125

---

* It is plain that $33\frac{1}{3}$ is equal to $\frac{100}{3}$, therefore any number which we would multiply by $33\frac{1}{3}$, is managed by annexing two cyphers first to the given multiplicand, and then dividing by 3. It is in fact nothing more than multiplying by $\frac{100}{3}$. This doctrine seems, however, to be better calculated for a place in fractions, than here.

QUESTIONS

TO EXERCISE THE FOREGOING RULES.

1. Of Simple Addition.

1. It was 73 years from the discovery of America, by Christopher Columbus, to the time of introducing potatoes into Ireland, from New Spain; 18 years from thence to the introduction of tobacco into England, from Virginia; 100 years from thence to the founding of Philadelphia. How long after the discovery of America, did the latter circumstance occur?

Ans. 191 years.

2. Bought 25 barrels of flour, at 13 dollars per barrel: Required, the price of the whole; also their weight; each being 196 pounds.

Ans. Price $325, weight 4900*lb.*

3. Thirty days are in September,
The same in April, June, November;
In February twenty-eight alone;
And all the rest just thirty-one.
Now sum them up and let us hear
How many days are in the year.

Ans. 365, except bissextile,* or leap
year, which has 366 days in it.

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* In every bissextile, or leap year, there is an additional day put to the month of February. This arises from the circumstance of our year containing 365 days and nearly six hours; these odd hours, in four years' time, amount to one day, and is accordingly placed to February. In a cycle of 130 years, the leap day may be thrown out of the reckoning; for allowing 365 days and 6 hours to each year, it is better than 11 minutes too much, which in 130 years amounts to a day too much.

4. If in 20 chests, be 20 drawers, in every drawer 20 purses, in every purse 20 dollars; how many purses and dollars do the 20 chests contain?

Ans. 400 purses, 8000 dollars.

5. An ox has two eyes, two ears, two horns, and four hoofs; in a drove of 20 oxen, how many eyes, ears, horns, and hoofs?

Ans. 40 eyes, 40 ears, 40 horns, and 80 hoofs.

6. What day of the year is the 29th day of October, new style?

Ans. 302d, but if bissextile, the 303d.

Of Simple Subtraction.

1. The specific gravity of platina rendered malleable,* is 20170 ounces Avoirdupoise, per cubic foot; very fine gold, 19637 ounces; required, the difference of their weights per foot.

Ans. 533 ounces.

2. How much further is the planet Georgian Sidus, which is 1565000000 miles from the sun, than our earth, which is only 95173000 miles distant?

Ans. 1469827000.

3. William Penn received a patent for Pennsylvania, A. D. 1680; how long is it since, reckoning to A. D. 1820? Ans. 140 years.

4. Christopher Columbus discovered America in A. D. 1492; how long is it since, reckoning to 1820?

Ans. 328 years.

5. The earth being distant from the sun 95173000 miles, and the moon distant from the earth 240000

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* Susceptible of being hammered.

miles, when the earth, moon, and sun form a conjunction,* at what distance from the sun will the moon be? Ans. 9493300 miles.

6. From 100 take 11, from what is left take 7, once again take 7 and 11; how much more than 7 and 11 shall we want to match it even?

Ans. 46.

Of Simple Multiplication.

1. Light is found to move 1230000 swifter than a cannon ball; now supposing the velocity of the ball to be 2000 feet per second, what is the velocity of light per second? Ans. 2460000000 feet.

2. The battering ram of Vespasian, which weighed 60000*lb.*, moving at the rate of 24 feet in one second, demolished the walls of Jerusalem. By the doctrine of forces, a cannon ball weighing 42*lb.*, would have to move at least 1428 times as swift, to do the same execution; what should the velocity of the 42 pounder be, to do as much execution as the ram? Ans. 34272 feet per second.

3. If a man was chopping at the distance of 380 yards from you, he could easily raise his axe after giving a stroke, before you could hear it: as sound moves about the above distance in one second of time, at what distance would a ship of war be from you, if you could just hear her morning gun 34 seconds after it was fired? Ans. 12920 yards.

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* A conjunction of the planets is formed, when two or more of them fall in a straight line on the same side of the sun, but in a line with it.

4. If a ship sails 9 knots an hour, and allowing 24 hours to the day and night; how far will she sail in 20 days and nights? Ans. 4320 knots, or miles.

5. What is the product of twenty score* multiplied by twenty gross? Ans. 96000.

6. Multiply 20 dozen of dozens by 11 score of dozens. Ans. 7603200.

Of Simple Division.

1. Took a prize at sea, valued at $24000. Now, suppose the officers, mariners and sailors agree to divide the money equally, and there are 500 men altogether; what is the share of each?

Ans. $48.

2. A piece of broad cloth, at $5 per yard, cost $375; how long was the piece? Ans. 75 yards.

3. How many charges for a mortar, each 13*lb.*, can be made out of 3341*lb.* of powder?

Ans. 257.

4. How many barrels, each 196*lb.*, can be packed out of 392000*lb.* of flour? Ans. 2000 bls.

5. If 5 yards of cloth make a suit for one soldier, how many may be clothed out of 12 bales, each 12 pieces, each 12 yards?

Ans. 345 soldiers, and 3 yds. over.

6. It is about 19008000 inches from Philadelphia to Pittsburg; how often will the wheel of a stage, which is 208 inches in circumference, turn round in running from one place to the other?

Ans. 91384 times, and 8 inches of a turn.

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* A score, equal 20.

COMPOUND ADDITION.

Compound Addition is the art of summing pounds, shillings, pence and farthings; hundreds, quarters, pounds, ounces, drams, &c. &c. or any thing else, where wholes and their parts are concerned.

OF ENGLISH MONEY.

1. The denominations of English money are as follow, viz.

4 farthings make 1 penny.
12 pence 1 shilling.
20 shillings 1 pound.

qrs. the contraction for *quadrantes* (farthings.)
d. *denarii*, or pence.
s. *solidi*, or shillings.
£. *libræ*, or pounds.

RULE.

1. Add the rank of farthings as whole numbers, divide their sum total by 4, (because four farthings make a penny,) set the remainder down for farthings, and add the quotient to the rank of pence.

2. Add up the rank of pence, divide the sum total by 12 (because twelve pence make a shilling,) set down the remainder for pence, and add the quotient to the rank of shillings.

3. Add the shillings in like manner; divide their sum total by 20 (because twenty shillings make one pound,) set down the remainder for shillings, and add the quotient to the rank of pounds.

4. Pounds being wholes, must be added as simple numbers, and the thing is done.

TABLE OF PENCE.

	And in addition we call	
¼=1 farthing,		¼ one.
½=2 farthings,		½ two.
¾=3 farthings,		¾ three.

PENCE.	SHILLINGS.	PENCE.	PENCE.	SHILLINGS.	PENCE.
12 make	1	0	130 make	10	10
20	1	8	140	11	8
30	2	6	150	12	6
40	3	4	160	13	4
50	4	2	170	14	2
60	5	0	180	15	0
70	5	10	190	15	10
80	6	8	200	16	8
90	7	6	210	17	6
100	8	4	220	18	4
110	9	2	230	19	2
120	10	0	240	20 or	£1.

EXAMPLES.

£	s.	d.	£	s.	d.	£	s.	d.
13	1	9	94	16	6	948	18	4¾
86	18	3	5	3	6	51	1	7¼
47	13	7	39	18	7	798	17	4½
52	6	5	60	1	5	201	2	7½
33	17	9	13	10	5	559	18	7
76	15	6	19	18	8	639	13	9¾

FEDERAL MONEY.

1. The denominations of Federal money are as follow, viz.

			dwt.	*gr.*
10 mills*	make	1 cent, copper, weight	11	00
10 cents		1 dime, silver,	1	17$\frac{3}{5}$
10 dimes		1 dollar, do.	17	7
10 dollars		1 eagle, gold,	11	6

ADDITIONAL COINS.

		dwt.	*gr.*
2. Half cent,	made of copper, weight	5	12
Half dollar	silver,	8	16
Quarter dollar,	do.	4	8
Half dime,	do.		22$\frac{4}{5}$
Half eagle,	gold,	5	15
Quarter eagle,	do.	2	19$\frac{1}{2}$

3. The *rule* for adding Federal money is evident; the denominations being in a decuple, or ten fold, ratio.

EXAMPLES.

E.	*D.*	*d.*	*c.*	*m.*
1748	4	7	8	9
8251	5	2	1	1
7472	6	8	7	3
2527	3	1	2	7
7386	7	3	6	9
3525	3	7	8	7

E.	*D.*	*d.*	*c.*	*m.*
1789	7	6	6	2
8210	2	3	3	8
7376	7	3	6	7
2623	2	6	3	3
6735	3	7	4	6
1386	3	4	6	7

---

* *Mill*, from *mille*, a thousand. *Cent*, from *centum*, an hundred. *Dime*, from *disme*, ten.

D.	c.	D.	c.	D.	c.
1673	75	6747	75	374	79½
8326	25	3252	25	625	20½
7346	87½	6743	87½	678	87½
2653	12½	3256	12½	321	12½
4756	37½	7467	62½	675	16
5678	59	8763	37½	874	69½

TROY[*] WEIGHT.

1. The denominations of Troy weight are as follow, viz.

24	grains make	1 pennyweight.
20	pennyweights	1 ounce.
12	ounces	1 pound.

2.

gr.	the contraction for grain.
dwt.	pennyweight.
oz.	ounce.
℔.	pound.

3. Troy weight is used for weighing gold,[†] silver, jewels, electuaries,[‡] and liquors.

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[*] Troy weight, from *Troyes*, a city in Champaign, from whence it came to be used in England.

[†] The standard for gold coin in the United States, is the same as in Great Britain, viz. Eleven parts fine, and one part alloy, (which is mostly copper.)
The standard for silver coin is 1485 parts of fine silver and 179 parts of alloy, which is, for most part, also copper.

[‡] Electuaries, a compound medicine, made of conserves and powders, about the consistence of honey.

RULE.

1. Add grains as simple numbers, divide their sum by 24, (because twenty-four grains make one pennyweight) set down the remainder for grains, and add the quotient to pennyweights.

2. Add pennyweights in like manner; divide their sum total by 20, because twenty pennyweights make one ounce,) set down the remainder for pennyweights, and add the quotient to ounces.

3. Add ounces the same way; divide their sum by 12, (because twelve ounces make one pound,) set down the remainder for ounces, and add the quotient to pounds.

4. Pounds being wholes, are added as simple numbers, and the thing is done.

EXAMPLES.

℔.	*oz.*	*dwt.*	*gr.*	℔.	*oz.*	*dwt.*	℔.	*oz.*	*dwt.*	*gr.*
27	10	17	21	84	10	16	16	8	10	11
72	1	2	3	15	1	4	83	3	9	13
83	9	10	10	75	10	17	67	4	10	12
16	2	9	14	24	1	3	32	7	9	12
75	11	17	15	76	11	17	75	11	17	15
16	9	27	16	15	10	16	37	10	16	16

AVOIRDUPOISE* WEIGHT.

1. The denominations of Avoirdupoise weight are as follow, viz.

16 drams make	1 ounce.
16 ounces	1 pound.
28 pounds	1 quarter of a hundred weight.
4 quarters	1 hundred weight, or 112℔.
20 hundred	1 ton.

2. *dr.* the contraction for	drams.
oz.	ounces.
℔.	pounds.
qr.	quarter of an hundred.
C.	hundreds.
T.	tons.

3. The use of Avoirdupoise weight is, to weigh all gross and drossy goods, groceries, and chandlers' wares; and all metals, except silver and gold.

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* An ounce Avoirdupoise, is to an ounce Troy, as 192 to 175; that is, an ounce Troy is equal to $1\frac{17}{175}$ ounces Avoirdupoise, and 175 Troy pounds are equal to 144*lb.* Avoirdupoise. The name Avoirdupoise is from the French, to have full weight. 5760 grains Troy, are equal 7000 grains Avoirdupoise.

A barrel of pork	220℔.	A puncheon, prunes	1120℔.
beef	220	A fother of lead	19*C.* 2*qr.*
A quintal of fish	112 Avoir.	A stone of shot	14℔.
A firkin of butter	56 do.	wool	16
soap	94 do.	A todd of wool	28
A barrel anchovies	30℔.	A wey	182
soap	256	A sack	364
raisins	112	A last	4368

RULE.

1. Add up drams as simple numbers; divide their sum total by 16, (because sixteen drams make an ounce,) set down the remainder for drams, and add the quotient to ounces.

2. Ounces must be added in like manner; divide their sum by 16, (because sixteen ounces make a pound,) set down the remainder for ounces, and add the quotient to pounds.

3. Pounds, add in the same way; divide by 28, (because twenty-eight pounds make a quarter of a hundred,) set down the remainder for pounds, and add the quotient to quarters.

4. Add quarters in like manner; divide the sum by 4, (because four quarters make a hundred,) set down the remainder for quarters, and add the quotient to hundreds.

5. Add hundreds in the same way; divide their sum by 20, (because twenty hundreds make a ton,) set down the remainder for hundreds, and add the quotient to tons.

6. Add tons as before; set the amount of the last rank all down and the thing is done.

EXAMPLES.

℔.	*oz.*	*dr.*
17	10	10
82	5	6
85	11	4
14	4	12
17	10	13
16	13	15

C.	*qr.*	℔.
18	2	25
81	1	3
77	2	18
22	1	10
39	2	23
17	3	22

T.	*C.*	*qr.*	℔.	*oz.*	*dr*
117	17	2	26	14	10
882	2	1	1	1	6
743	10	2	7	14	11
256	9	1	20	1	5
643	17	2	10	12	14
475	18	1	19	13	15

APOTHECARIES WEIGHT.

1. The denominations of Apothecaries weight are as follow, viz.

20 grains	make	1 scruple.
3 scruples		1 dram.
8 drams		1 ounce.
12 ounces		1 pound.

2. ℔ the contraction for	pounds,	or ℔.
℥	ounces,	*oz.*
ʒ	drams,	*dr.*
℈	scruples,	*sc.*
gr.	grains,	*gr.*

3. The Apothecaries' pound and ounce are the same as the pound and ounce Troy, and is only used for compounding or mixing medicine.

RULE.

1. Add grains as simple numbers; divide their sum by 20, (because twenty grains make a scruple,) set down the remainder for grains, and add the quotient to scruples.

2. Let scruples be added in like manner; divide their sum by 3, (because three scruples make a dram,) set down the remainder for scruples, and add the quotient to drams.

3. Add as before; divide their sum by 8, (because eight drams make an ounce,) set down the remainder for drams, and add the quotient to ounces.

4. Add ounces the same way; divide their sum by 12, (because twelve ounces make a pound,) set the remainder down for ounces, and add the quotient to pounds.

5. Pounds are added as simple numbers, and the thing is done.

EXAMPLES.

℔.	*oz.*	*dr.*
82	10	6
17	1	2
37	8	2
62	3	6
75	10	5
83	9	7

℔.	*oz.*	*dr.*	*sc.*	*gr.*
75	11	7	2	12
24	00	0	0	8
69	10	6	1	16
30	1	1	1	4
40	8	6	2	16
39	6	5	2	19

dr.	*sc.*	*gr.*
4	2	11
5	0	9
7	1	8
2	1	12
6	2	17
5	1	16

LONG MEASURE.

1. The denominations of Long Measure are as follow, viz.

3	barleycorns make	1 inch.
4	inches	1 hand.
12	inches	1 foot.
3	feet	1 yard.
6	feet	1 fathom.
5½	yards	1 rod, pole, or perch.
40	poles	1 furlong.
8	furlongs	1 mile.
3	miles	1 league.
60	geographic miles	1 degree.
360	degrees	1 whole circle.
69½	English miles	1 degree.

And 360 × 69½ = 25020 miles, the whole great circle of the earth.

2. *bc.* the contraction for barleycorns. *in.* for inches. *ft.* for feet. *fath.* for fathoms. *yds.* for yards.

p. for poles. *f.* for furlongs. *m.* for miles. *L.* for leagues. *D.* for degrees.

3. The use of Long Measure is, to ascertain the distances of places, length of timber, buildings, &c.

4. The hand, is to measure the heighth of horses.

5. The fathom, to measure depths.

6. The pole or perch, to measure land, &c.

7. The mile, to measure roads.

8. Leagues, to measure distances at sea, sometimes on land.

9. Degrees, to measure latitudes and longitudes.

10. Inches and feet; carpentry, &c. and the yard, cloth, &c.

NOTE. There are two and four pole measuring chains used by surveyors, consisting of 50 and 100 links, of 7.92 inches each; making them 33 and 66 feet long. Sometimes chains of the above lengths, contain but 40 and 80 links, of 9.9 inches each, which are handier, because two links make the tenth of a perch; of the former, two and a half.

A geometrical pace, 5 feet. A line, 6 points. 12 lines, an inch. A French toise, 6*f.* 4*in.* 8.8 lines English.

RULE.

1. Add barleycorns as simple numbers; divide their sum by 3, (because three barleycorns make an inch,) set down the remainder for barleycorns, and add the quotient to inches.

2. Add inches in like manner; divide their sum by 12, (because twelve inches make a foot,) put down the remainder for inches, and add the quotient to feet.

3. Add feet the same way; divide their sum by 3, (because three feet make a yard,) set down the remainder for feet, and add the quotient to yards.

4. Add yards in like manner; divide their sum by 5½, or twice their sum by 11; set down the remainder for yards, and add the quotient to poles.

5. Add each rank which follows in order, in manner aforesaid; divide and set down, as the denomination directs, to the last, and the thing is done.

EXAMPLES.

yds.	*ft.*	*in.*
5	2	11
4	0	1
3	1	7
6	1	5
2	0	2
7	2	10
4	2	11
5	2	9

yds.	*ft.*	*in.*	*bc.*
7	2	6	1
2	0	5	2
4	1	10	2
5	1	1	1
3	2	8	1
6	0	3	2
7	2	11	1
6	2	9	2

L.	*m.*	*fu.*	*p.*	*yds.*	*ft.*	*in.*	*bc.*
12	2	5	16	4	2	6	1
87	0	2	23	½	0	5	2
61	1	3	20	3½	1	7	2
38	1	4	19	1	1	4	1
16	2	6	28	3	0	10	1
33	0	1	11	1½	2	1	2
75	2	7	27	3	2	11	1
67	2	6	29	4½	1	11	2

CLOTH MEASURE.

1. The denominations of Cloth Measure are as follow, viz.

2¼ inches make	1 nail.
4 nails	1 quarter of a yard.
4 quarters	1 yard.
3 quarters	1 ell Flemish.
5 quarters	1 ell English.
6 quarters	1 ell French.
4 qrs. 1⅕ in.	1 Scotts' ell.
3⅔ quarters	1 Spanish var.

RULE.

1. Add inches as whole numbers; divide by 2¼, (because two and a quarter inches make a nail,) set down the remainder for inches, and add the quotient to nails.

2. Nails are added in like manner, and divided by 4, (because four nails make the quarter of a yard,) set down the remainder for nails, and add the quotient to quarters.

3. Add quarters the same way; divide their sum by 4, (because four quarters make a yard,) set down the remainder for quarters, and add the quotient to yards. If ells English, divide by 5; Flemish ells, by 3; and French ells, by 6. The last name being added as simple numbers, and all set down, completes the operation.

4. Contraction for inches, *in.* nails, *na.* quarters, *qr.* yards, *yds.* ells English, *E.E.* ells Flemish, *E.F.* ells French, *E.Fr.*

EXAMPLES.

yds.	*qr.*	*na.*
33	2	2
66	1	2
17	1	1
82	2	3
75	1	1
24	2	3
79	3	3
65	3	2

E.E.	*qr.*	*na.*
39	4	3
60	0	1
84	1	1
15	3	3
75	2	2
24	2	2
79	3	3
87	4	3

E.F.	*qr.*	*na.*	*in.*
75	2	3	2
24	0	0	¼
66	1	2	1½
33	1	1	¾
85	1	2	1
14	1	1	1¼
67	2	3	1
49	1	3	2

LAND MEASURE.

1. The denominations of Land Measure are as follow, viz.

9 square feet make 1 square yard.
30¼ yards 1 square perch.
40 perches 1 rood.
4 roods, or 160 *p.* 1 acre.

2. A square yard, perch, &c. may be understood as being the same length and breadth.

3. The use of square measure is, to ascertain the content of land, or any thing else, consisting of length and breadth.

RULE.

1. Add square feet as simple numbers; divide their sum by 9, (because nine square feet make one yard,) set down the remainder for feet, and add the quotient to yards.

2. Add yards in like manner; divide their sum by 30¼, (because thirty and a quarter square yards make one square perch,) set down the remainder for yards, and add the quotient to perches.

3. Add these as before; divide their sum by 40, (because forty perches make one rood,) set down the remainder for perches, and add the quotient to roods.

4. Add roods as simple numbers; divide their sum by 4, (because four roods make an acre,) set down the remainder for roods, and add the quotient to acres.

5. Acres, add in like manner; set all down, and the thing is done.

ft. contraction for feet, *yds.* for yards, *P.* for perches, *R.* for roods, *A.* for acres.

EXAMPLES.

A.	*R.*	*P.*
17464	2	36
82535	1	4
74342	2	18
25657	1	22
75474	3	37
67474	3	19

A.	*R.*	*P.*
375	3	17
624	0	23
743	2	20
256	1	20
743	3	33
674	1	37

A.	*R.*	*P.*	*yds.*	*ft*
174	2	30	27	6
825	1	9	2¼	3
666	2	27	20	4
333	1	12	9¼	5
746	2	33	27½	4
674	3	39	17¾	7

SOLID MEASURE.

1. the denominations of Solid Measure are as follow, viz.

64 solid or cubic quarters make	1 inch.
1728 inches	1 foot.
27 feet	1 yard.
40 feet of round timber, or 50 feet of square	1 load or ton.
128 cubic feet (8 feet long, 4 feet high, and 4 feet broad)	1 cord of wood or bark.

2. The use of Solid Measure is, to ascertain the content of timber, stone, masons' work, &c. and whatever has length, breadth and thickness.

3. A solid inch, foot, yard, &c. is an inch, foot, or yard long, broad, and deep.

Rule here omitted.

yds.	*ft.*	*in.*	*qr.*	*Tons.*	*ft.*	*in.*	*Cords.*	*ft.*	*in.*	*qr.*
267	20	1675	50	39	30	1700	1370	100	1700	57
732	6	52	14	60	9	28	8629	27	27	7
666	18	743	58	45	17	1600	7432	54	160	40
333	8	984	11	54	22	128	2567	73	1557	24
674	23	749	17	75	17	1544	7467	76	174	30
674	25	1174	60	67	19	1720	8742	69	1147	60

WINE MEASURE.

1. The denominations of Wine Measure are as follow, viz.

2	pints	make	1 quart.
4	quarts		1 gallon.
10	gallons		1 anchor of brandy.
18	gallons		1 rundlet.
31½	gallons		1 barrel.
42	gallons		1 tierce.
63	gallons		1 hogshead.
2	hogsheads		1 pipe or butt.
2	pipes		1 ton.

2. The use of Wine Measure is, to ascertain the content of brandy, spirits, perry, cyder, honey, oil, and some other liquids.

RULE.

1. Add pints as simple numbers; divide their sum by 2, (because two pints make one quart,) set down the remainder for pints, and add the quotient to quarts.

2. Add these in like manner; divide their sum by 4, (because four quarts make one gallon,) set down the remainder for quarts, and add the quotient to gallons.

3. So proceed with the remaining names, still dividing as before, until the whole operation is performed.

pts. contraction for pints. *qts.* for quarts. *gal.* for gallons. *An.* for anchor. *Run.* for rundlet. *B.* for barrel. *Tie.* for tierce. *Hhd.* for hogsheads. *Pi.* for pipes. *T.* for tons.

NOTE. A pint measure for wine, &c. contains 28⅜ cubic inches; quart, 57¾; gallon, 231. For beer or ale, the pint contains 35¼ cubic inches; the quart, 70½; and the gallon, 282.

NOTE. 2. *Rules* for the following cases in Compound Addition omitted, as they may be easily gathered from the tables.

EXAMPLES.

Tie.	*gal.*	*qt.*	*pt.*
19	38	3	1
80	3	0	1
76	18	1	1
23	23	2	1
76	41	3	1
94	39	3	1

Hhd.	*gal.*	*qt.*
75	31	2
24	31	2
67	34	1
32	28	3
74	37	2
84	61	3

T.	*Hhd.*	*gal.*	*qt.*	*pt.*
8	2	32	2	1
1	1	30	1	1
8	1	26	1	1
1	2	36	2	1
7	3	34	2	1
6	3	61	3	1

ALE AND BEER MEASURE.

1. The denominations of Ale and Beer Measure are as follow, viz.

2 pints	make	1 quart.
4 quarts		1 gallon.
8 gallons		1 firkin of ale in London.
8½ gallons		1 firkin of beer.
9 gallons		1 firkin of beer in London.
2 firkins		1 kilderkin.
1½ barrels, or 54 gal.		1 hogshead of beer.
2 barrels		1 puncheon.
3 barrels, or 2 Hhd.		1 butt.

2. *Hhd.* contraction for hogsheads. *Fir.* firkins. *Kil.* kilderkins. *Ab.* ale barrels. *Bb.* beer barrels. *Pu.* puncheons. *Bu.* butts.

3. The use of Ale and Beer Measure is, to buy and sell those liquors by it.

EXAMPLES.

Ab.	*fir.*	*gal.*	*Hhd.*	*gal.*	*qt.*	*Bb.*	*gal.*	*qt.*	*Bb.*	*fir.*	*gal.*
30	2	6	307	40	2	44	30	2	742	3	3
69	1	2	692	13	2	55	5	2	257	0	1
45	2	4	174	25	1	73	16	1	643	2	4
54	1	4	825	28	3	26	19	3	356	1	4
66	3	7	746	33	3	74	33	3	746	0	4
74	3	6	474	39	2	64	35	2	694	1	6

DRY MEASURE.

2	pints	make	1 quart.
2	quarts		1 pottle.
2	pottles		1 gallon.
2	gallons		1 peck.
4	pecks		1 bushel.
2	bushels		1 strike.
2	strikes		1 coom.
2	cooms		1 quarter.
4	quarters		1 chaldron. *Ch.*
4½	quarters		1 chaldron, London.
5	quarters		1 wey.
2	weys		1 last.

2. The use of Dry Measure is, to ascertain the quantity of corn, seed, fruit, roots, salt, sand, coals, &c. for the purpose of buying and selling them.

3. *pot.* contraction for pottles; *pe.* for pecks; *bu.* for bushels; *str.* for strikes; *q.* for quarters; *co.* for cooms; *we.* for weys; *la.* for lasts.

4. The Dry Measure gallon contains $268\frac{4}{5}$ cubic inches; the bushel, $2150\frac{2}{5}$. A cylindrical measure of 18 inches wide and 8 inches deep, will hold a bushel.

EXAMPLES.

pot.	*qt.*	*pt.*	*bu.*	*pe.*	*gal.*	*qt.*	*q.*	*bu.*	*pe.*	*Ch.*	*bu.*	*pe.*	*gal.*
5	3	1	39	2	1	3	74	6	2	78	30	2	1
4	4	1	60	1	0	1	25	1	2	21	5	1	1
3	6	1	72	2	1	2	67	3	3	49	16	2	1
6	1	1	27	1	0	2	32	4	1	50	19	1	1
3	3	1	44	2	1	3	76	6	3	74	33	2	1
4	1	6	49	2	1	3	79	7	3	78	35	2	1

TIME.

1. The denominations of Time are as follow, viz.

60 seconds	make	1 minute.
60 minutes		1 hour.
24 hours		1 day.
7 days		1 week.
4 weeks		1 month.
13 months, 1 day, 6 hours		1 Julian year.

2. *365 days, 6 hours, 9 minutes, 15½ seconds, one siderial year.

3. 365 days, 6 hours, one Julian year.

4. 365 days, 5 hours, 58 minutes, 57½ seconds, one solar year.

Measures, or divisions of Time, are used for purposes of navigation, history, commerce, &c.

sec. contraction for seconds; *mi.* for minutes; *h.* for hours; *d.* for days; *w.* for weeks; *M.* for months; *Y.* for years.

EXAMPLES.

d.	*h.*	*mi.*	*sec.*
6	12	30	30
3	11	29	30
5	9	36	36
4	14	23	24
6	11	39	59
4	12	49	59

w.	*d.*	*h.*
3	4	12
6	2	12
4	5	13
5	1	11
6	4	19
4	6	23

Y.	*M.*	*w.*	*d.*	*h.*
174	8	2	4	12
829	3	1	2	12
743	2	1	2	18
256	9	2	4	6
743	11	3	6	23
174	10	2	6	22

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* 1. *Siderial year;* the time which the earth, in her orbit, requires to move from any fixed star, until she returns to the same.

2. *Julian year;* a period fixed by Julius Cesar, one of the Roman emperors, otherwise called the civil year.

3. *Solar year;* the time in which the earth performs one revolution round the sun.

MOTION.

1. The denominations of Motion are as follow, viz.

60 seconds	make	1 prime, minute.
60 minutes		1 degree.
30 degrees		1 sign.
12 signs, or 360 deg.		1 circle of the sphere.

2. The above, are measures of the zodiac,* &c. and are used in various astronomical computations, as eclipses, solar and lunar; transits, &c.

3. ″ mark to designate seconds; ′ minutes; ° degrees; *sig.* signs; *cir.* circles.

EXAMPLES.

°	′	″
44	36	36
55	23	24
14	30	30
85	29	30
74	39	49
17	38	54

sig.	°	′	″
11	20	30	30
88	9	29	30
75	15	34	39
24	14	25	21
37	27	19	39
18	25	59	48

cir.	*sig.*	°	′	″
34	8	24	50	50
65	3	5	9	10
47	10	16	29	39
52	1	13	30	21
74	9	19	39	49
14	9	29	49	59

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* Names of the signs of the zodiac.

The sun enters

Aries	♈	March	20	Libra	♎	Sept.	22
Taurus	♉	April	19	Scorpio	♏	Oct.	22
Gemini	♊	May	20	Sagittarius	♐	Nov.	21
Cancer	♋	June	21	Capricorn	♑	Dec.	21
Leo	♌	July	22	Aquarius	♒	Jan.	19
Virgo	♍	August	22	Pisces	♓	Feb.	13

COMPOUND SUBTRACTION.

1. Compound Subtraction is the art of finding the difference between any two quantities of the same kind, and their parts, composed of different names, as, pounds, shillings, pence, &c.

2. Place the given quantities so, that like names may stand under each other; the greater quantity and its parts, above the lesser and its parts.

RULE.

1. If the upper quantity and all its parts, be greater than the lower one and its parts, subtract as in simple quantities.

2. If some of the parts of the lesser quantity be greater than those above them, so much must first be added to such upper part, as make one of the next higher denomination; then subtract, note the difference, and add one to the next higher rank.

3. Proceed from rank to rank; observing the same precaution to the last, and the thing is done.

EXAMPLES.

OF ENGLISH MONEY.

£	s.	d.	£	s.	d.	£	s.	d.	qrs.
1744	16	6	6174	18	4	175	18	3	½
302	13	2	1030	19	7	95	18	3	¾

FEDERAL MONEY.

D.	*d.*	*c.*
74	9	7
40	9	8

D.	*d.*	*c.*	*m.*
490	8	7	4
300	8	9	5

D.	*c.*
74	62½
40	37¾

E.	*D.*	*d.*	*c.*	*m.*
40	3	4	7	2
10	2	9	8	7

TROY WEIGHT.

℔	*oz.*	*dwt.*
14	11	17
7	4	11

℔	*oz.*	*dwt.*	*gr.*
19	4	17	22
9	1	18	23

oz.	*dwt.*	*gr*
10	19	22
9	19	23

AVOIRDUPOISE WEIGHT.

℔	*oz.*	*dr.*
12	14	14
9	15	15

C.	*qr.*	℔
17	3	27
9	3	10

T.	*C.*	*qr.*	℔	*oz.*	*dr*
174	17	2	23	14	14
44	17	2	27	14	15

APOTHECARIES WEIGHT.

℥	ʒ	℈	*gr.*
10	7	2	11
4	7	2	18

℔	℥	ʒ
17	10	7
6	10	1

℔	℥	ʒ	℈	*gr.*
14	10	4	2	17
4	10	5	2	18

LONG MEASURE.

ft.	*in.*	*bc.*
19	7	2
9	8	0

yds.	*ft.*	*in.*	*bc.*
5	2	11	1
4	2	11	2

L.	*m.*	*fu.*	*p.*	*yd.*	*ft.*	*in.*	*bc.*
60	2	7	39	4	2	10	1
17	2	7	39	6	2	11	2

LAND MEASURE.

A.	*R.*	*P.*
1000	1	34
177	1	37

A.	*R.*	*P.*
474	8	17
19	3	29

A.	*R.*	*P.*
176	1	14
19	2	39

SOLID MEASURE.

yds.	*ft.*	*in.*
100	0	00
		11

Cor.	*ft.*	*in.*
40	100	1110
1	124	1117

Tons.	*ft.*	*in.*
40	37	1000
2	37	1009

yds.	*ft.*	*in.*
60	8	1007
17	8	1101

WINE MEASURE.

Tie.	*gal.*	*qt.*
67	50	3
6	41	1

Hhd.	*gal.*	*qt.*
175	60	2
69	62	3

T.	*Hhd.*	*gal.*	*qt.*	*pt.*
167	3	60	2	1
94	3	61	3	0

ALE AND BEER MEASURE.

Ab.	*fir.*	*gal.*
64	3	7
16	3	1

Bb.	*fir.*	*gal.*
167	2	7
84	2	8

Ahd.	*gal.*	*qts.*	*pts.*
640	49	2	1
99	52	1	0

DRY MEASURE.

bu.	*pe.*	*gal.*
179	3	1
99	8	0

bu.	*pe.*	*ga,*	*qt.*
198	3	1	2
74	3	1	3

Ch.	*bu.*	*pe.*
148	32	3
64	33	0

Qrs.	*bu.*	*pe.*
174	4	1
84	7	3

TIME.

h.	*m.*	*sec.*
17	39	49
0	39	59

d.	*h.*	*m.*	*sec.*
4	20	49	42
3	21	50	50

Y.	*M.*	*w.*	*d.*	*h.*	*m.*	*sec.*
184	10	2	6	20	17	16
88	11	3	6	21	18	17

MOTION.

°	′	″
14	48	49
9	19	58

sig.	°	′	″
10	20	49	59
7	27	49	10

cir.	*sig.*	°	′	″
147	10	14	54	48
84	9	10	17	59

QUESTIONS

TO EXERCISE COMPOUND SUBTRACTION.

1. Subtract $\frac{1}{4}$ from £1000. Ans. £999 19 11$\frac{3}{4}$.

2. Subtract 1 mill from 100 eagles.
Ans. 99, 9, 9, 9, 9.

3. Subtract 1 barleycorn from a mile.
Ans. 7*f.* 39*p.* 4*yds.* 2*ft.* 11*in.* 2*bc.*

4. Subtract 1 second from 1 year.
Ans. 11*M.* 3*w.* 6*d.* 23*h.* 59*m.* 59*sec.*

5. A man born in the year A. D. 1776, April 1, how old is the person on the 2d day of January, 1821? Ans. 44*Y.* 9*M.* 1*d.*

6. Drew out of a deposit in bank, amounting to $1794,44, at one time, $500; at three several times, $375 each time; gave a check for $79,14 to A.; on settling my account, what balance had I to lift?
Ans. $90, 30.

7. How long is it from the 19th of January, 1811, to the 31st day of March, 1821? Ans. 10*Y.* 2*M.* 12*d.*

8. How many years is it from the 11th of June, 1799, to Christmas, 1821? Ans. 22*Y.* 6*M.* 14*d.*

COMPOUND MULTIPLICATION.

1. Compound Multiplication is a concise way of performing extensive additions of numbers and their parts, of divers names; as, pounds, shillings, &c.

2. Compound Multiplication is used for casting up the price of any number of pounds, yards, acres, &c. at any given rate.

RULE.

1. Begin at the lowest part of the given quantity.

2. Multiply such part by the given multiplier, as in simple multiplication; divide the product by such number as it requires of said part to make one of the next higher.

3. Set down the remainder, and add the quotient to the next higher place.

4. Proceed in like manner with each part successively to the last or whole part, which must be proceeded with as simple numbers, and the thing is done.

EXAMPLES.

OF ENGLISH MONEY.

Multiply	£1	14	6	by 2	Ans.	£3	9	0
	3	17	4	by 3=		11	12	0
	4	19	6¼	by 4=		19	18	1

Multiply	£7 18 9 by 5=	Ans.	£39 13 9
	17 18 4½ by 6=		107 10 3
	74 17 6¾ by 7=		524 2 11¼
	94 00 7½ by 8=		752 5 0
	79 18 4¼ by 9=		719 5 2¼

Multiply	$475, 44 by 13=*	Ans.	$6180, 72
	74, 75 ×14=		1046, 50
	749, 62½ ×15=		11244, 37½
	94, 79 ×16=		1516, 64
	748, 67 ×17=		12727, 39
	1474, 87½ ×18=		26547, 75
	119, 37½ ×19=		2268, 12½
	174, 67¼ ×20=		3493, 45

Multiply 16*lb.* 11*oz.* 10*dwt.* by 2=
1*qr.* 17 14 ×3=
16*C.* 3 24 15 14*dr.* ×7=
20*T.* 17 2 27 14 10 ×9=
19℔ 11℥ 7ʒ 2℈ 19*gr.* ×7=
40*L.* 2*m.* 7*f.* 30*p.* 4*yds.* 2*ft.* 11*in.* 2*bc.* ×11=
40*E.E.* 4*qr.* 3*na.* 2*in.* ×7=
30*E.F.* 2 2 1 ×11=
44 1 3 2 ×12=
364*A.* 3*R.* 39*P.* ×3=
744 2 36 ×7=
740*yds.* 8*ft.* 1164*in.* ×9=
370 *cords.* 100 1714 ×4=

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* When multiplication is performed of dollars and cents, two places must be always cut off to the right for cents. If dollars, cents and mills be multiplied, then three places must be cut off to the right hand; the outside one will be mills, the two inner ones, cents.

Multiply 40*Tie.* 41*gal.* 3*qt.* 1*pt.* × 11=
90*Hhd.* 54 3 1 × 7=
34*Bb.* 30 3 1 × 5=
37*Ab.* 29 2 1 × 9=
49*bu.* 3*pe.* 1*gal.* × 11=
37*Ch.* 30*bu.* 3*pe.* × 12=
84*q.* 7*bu.* 2*pe.* 1*pt.* × 5=
45*Y.* 11*M.* 3*w.* 6*d.* 23*h* 6*mi.* 59*sec.* × 3=
3*cir.* 11*sig.* 29° 47′ 44″ × 7=

When the multiplier happens to be a composite number, multiply by the component parts, instead of the number itself.*

		£	*s.*	*d.*
What cost 14 yards of calico at		0	3	4¼
4×4=16	do.		5	7½
4×5=20	do. broad cloth	1	17	6
3×7=21	do.	2	12	6
4×6=24	do.	3	15	0
5×5=25*lb.*	indgo,	1	2	6
6×6=36*C.*	iron,	$5, 37½		
10×4=40*T.*	hemp,	200, 62½		
9×5=45	barrels flour,	11, 34		
7×7=49	bushels wheat,	1, 37½		
10×5=50	boxes tin,	17, 00		
9×6=54	gallons varnish,	4, 35		
8×7=56	boxes 8 by 10 glass,	13, 39		
8×8=64	cases gin,	21, 75		
9×9=81	boxes Spanish cigars,	9, 25		
10×10=100	cases fine razors,	4, 29		
11×11=121	doz. pair stockings,	24, 60		

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* The reason for multiplying by the component parts of a number, instead of the number itself, is, that you save the operation of addition in all such cases.

Multiply £474 16 9½ × 5 by 3795.

		10	
4748	7	11	× 9
		10	
47483	19	2	× 7
		10	
474839	11	8	
		3	
1424518	15	00	= 3000 times £474 16 9½
332387	14	2	= 700 do. do.
42735	11	3	= 90 do. do.
2374	3	11½	= 5 do. do. and
			3000+700+90+5=3795
£1802016	4	4½	Ans. the given multiplier.

Note. Any sum and its parts, whether it be English money or any thing else, may be conveniently multiplied as above, by any multiplier, however great.

What comes 149 tons of pig iron to, at $37 50?
175 barrels shad, at £4 13 4½?
5749*lb.* soal leather, at 23½ cts.?
4094*lb.* flax, at 2*s.* 6*d.* ½?
1174 bushels flax seed, at 93½ cts.?
1449 bushels Indian corn, at 62½ cts.?
666*lb.* coffee, at 31¼ cts.?
104740*ft.* boards, at $8 62½ per thousand?
10047 shingles, at $10 37½ per thousand?
5000 rails, at $5 50 per hundred?
4746947474*ft.* scantling, at $2 12½ per thous.?

COMPOUND DIVISION.

Compound Division is the art of performing, in a concise manner, extensive subtractions of numbers, of divers names; as, pounds, shillings, pence, &c.

RULE.

1. When the divisor is less than 12, the operation is performed by Short Division; first measuring the highest name given, by the divisor.

2. Note the quotient underneath; if any thing be left, reduce it to the next lower name, if such name be also given, add the two together, and divide as before.

3. Continue the operation to the last, still carrying forward the respective remainders, properly reduced, adding and dividing as before, and the thing is done.

4. If the divisor be greater than 12, use Long Division, first measuring the largest given part; reduce the remainder to the next lower name, add such parts to it as may be given, divide as before; so proceed to the last, and the thing is done.

EXAMPLES.

Divide	£2 14 6	by 2.	Ans.	£1 7 3
	5 19 6½	3.		1 19 $10\frac{1}{6}$
	37 14 9	5.		7 10 $11\frac{2}{5}$
	167 19 6½	7.		23 19 $11\frac{3}{14}$
	$3757,62½	8.		$469, 70 3+
	1746, 37½	9.		194, 4 1+
	87469, 60	11.		7951, 78 1+
	167464, 80	12.		13955, 40

Divide 47*T*. 17*C*. 3*qrs*. 20*lb*. by 7.

100 19 2 27 11.

Divide 400*T.* 3*Hhd.* 60*gal.* 3*qt.* by 12.

Divide 7000*A.* 3*R.* 30*P.* into 11 equal shares.

744 2 16 into 9 shares.

1. Divide 1000 yards of linen amongst 30 persons.

2. How many parcels of 19 each can be made of 6859 things?

3. How many dozen of dozens may be made of 17840 dozens?

4. A captain and crew of sailors took a prize at sea; amongst other booty they fell upon a chest which contained 344 moidores, English currency, 751 guineas, and 955 Spanish dollars. The captain and sailors made 20 in number; but he alone was to have $\frac{5}{12}$ parts of the whole, the rest was to be equally divided amongst the sailors, which were of course 19 in number: required, the number of pounds that fell to the share of each.

Ans. Captain £611 11*s.* 10½*d.*
Each sailor 45 1 4+

5. If 3½ yards of cloth make a suit for one soldier, how many will 1700 yards clothe?

Ans. 485, and 2½ *yds.* over.

6. How many rations of 7*lb.* each can be made out of 1610*lb.* of beef? Ans. 230.

7. Suppose a maid carrying apples to market was met by three boys, and that the first took half what she had, but returned her back 10; that the second took ⅓, but returned 2; lastly, the third took ½ those she had left, but returned her 1; when she got clear, she had 12 apples left. What number had she at first? Ans. 40.

Divide £37467 19 6 amongst 39 persons.

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Operation. 39)37467 19 6(960 11 9¼ & 7/13. Ans.
              351
              ---
               236
               234
               ---
                 27
                  20 I here reduced the remainder
                 ---        to shillings, and brought
               39)559(11 in the odd ones.
                  39
                  ---
                  69
                   39
                   --
                   30
                   12 Here I reduced the remain-
                  ---      der to pence, and brought
               39)366(9  in the odd ones.
                  351
                  ---
                   15
                    4 Here I reduced the remain-
                   --      der to farthings.
                39)60(1
                   39
                   --
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$\frac{21}{39}=\frac{7}{13}$

In the above manner, any number of pounds, shillings, pence, &c. can be divided into as many parts as we please.

1. How many portions of $750 each can be paid out of $1000000?
2. How many portions of 333\frac{1}{3}$ can be paid out of $250000?
3. How many crowns and dollars, of each an equal number, may be counted out of $21000?

REDUCTION.

1. Reduction is the art of changing quantities from one denomination to another.

2. Quantities are changed from one denomination, or name, to another, by multiplication and division.

3. Multiplication is used to reduce higher to lower names; as, pounds to shillings, shillings to pence, &c.

4. Division is used to bring lower to higher names; as, farthings to pence, pence to shillings, &c.

5. Operations in which multiplication is used, are called descending; and those in which division is used, ascending.

REDUCTION DESCENDING.

RULE.

1. Multiply the highest part of the given quantity by as many of the next inferior part as make one of the first, to this product add such parts of the second name as are given.

2. Multiply again, by as many of the third part, or name, as make one of the second, adding any third parts that may be given, as before; so proceed from name to name, to the whole is gone over, and the thing is done.

EXAMPLES.

1. In 50 eagles, how many dollars, dimes, cents, and mills? Ans. $500, 5000*d.* 50000*cts.* 500000*m.*

2. In £50, how many shillings, pence, and farthings? Ans. 1000*s.* 12000*d.* 48000*qrs.*

3. In £11 19 11¾, how many farthings?
Ans. 11519*qrs.*

4. In 474½ eagles, how many mills?
Ans. 4745000*m.*

6. In £1000, how many half pence?
Ans. 480000 *half pence.*

TROY WEIGHT.

1. In 20*lb.* how many grains? Ans. 115200*gr.*
2. In 10*lb.* 10*oz.* 10*dwt.* how many grains? Ans. 62640*gr.*
3. In 100*lb.* 11*oz.* 19*dwt.* 23*gr.* how many ounces, pennyweights, and grains? Ans. 1211*oz.* 24239*dwt.* 581759*gr.*

AVOIRDUPOISE WEIGHT.

1. In 20 tons, how many pounds? Ans. 44800*lb.*
2. In 9*T.* 19*C.* 3*qrs.* 19*lb.* how many ounces? Ans. 358256*oz.*
3. In 10*T.* 19*C.* 3*qrs.* 19*lb.* 15*oz.* 15*dr.* how many drams? Ans. 6305791*dr.*

APOTHECARIES WEIGHT.

1. In 3*lb.* how many ounces, drams, scruples, and grains? Ans. 36*oz.* 288*dr.* 864*sc.* 17280*gr.*
2. In 10*lb.* 3*oz.* 7*dr.* 1*sc.* 19*gr.* how many grains? Ans. 59499*gr.*

LONG MEASURE.

1. In 9 leagues, how many inches? Ans. 1710720*in.*
2. In 7*L.* 2*m.* 7*f.* 39*p.* 4½*yds.* 2*ft.* 11*in.* how many inches? Ans. 1520639*in.*
3. In 100*L.* how many barleycorns? Ans. 57024000*bar.*

CLOTH MEASURE.

1. In 20 yards, how many nails? Ans. 320*na.*
2. In 40*yds.* 3*qrs.* 3*na.* how many inches? Ans. 1473¾*in.*
3. In 60 yards, how many ells Flemish? Ans. 80*E.F.*

4. In 1000 yards, how many ells English?

Ans. 800*E.E.*

5. In 40 ells English and 50 ells French, how many inches?

Ans. 4500*in.*

6. In 20 Scotch ells, how many inches?

Ans. 744*in.*

LAND MEASURE.

1. In 400 acres, how many square perches?

Ans. 64000*P.*

2. In 350*A.* 3*qrs.* 39*P.* 15*ft.* 100*in.* how many square inches?

Ans. 2201659696*in.*

SOLID MEASURE.

1. In 40 feet of round timber, how many solid inches?

Ans. 86400*in.*

2. In a cellar 30 feet long, 25 feet wide, and 9 feet deep, how many cubic feet?

Ans. 6750*ft.*

3. In a canal 5000 yards long, 20 yards wide, and 3 yards deep, how many cubic yards?

Ans. 300000*yds.*

4. If a stone wall be 200 feet long, 2 feet thick, and 15 feet high, how many cubic feet?

Ans. 6000*ft.*

5. The reservoir at the water works on Schuylkill, near Philadelphia, is 317 feet long, 168 feet wide, and 9 feet deep, how many cubic feet does it contain?

Ans. 479304 cubic feet.

WINE MEASURE.

1. In 5 tons of wine, how many pipes, hogsheads, gallons, quarts and pints?

Ans. 10*Pi.* 20*Hhd.* 1260*gal.* 5040*qts.* 10080*pts.*

2. In 30*Hhd.* 3*gal.* 3*qts.* 1*pt.* how many pints?

Ans. 15151*pts.*

3. In one ton, how many cubic inch measures?

Ans. 58212.

ALE AND BEER MEASURE.

1. In 10 barrels of ale, how many firkins, gallons, and quarts? Ans. 40*fir.* 320*gal.* 1280*qts.*

2. In 5*B.* 3*fi.* 7*gal.* 3*qts.* 1*pt.* how many pints? Ans. 1535*pts.*

3. In 10 hogsheads of beer, how many cubic inch measures. Ans. 152280.

DRY MEASURE.

1. In 100 bushels of wheat how many pecks, gallons, quarts, and pints?
Ans. 400*pe.* 800*gal.* 3200*qts.* 6400*pts.*

2. In 30*q.* 7*bu.* 3*pe.* 0*gal.* 3*qt.* 1*pt.* how many pints? Ans. 15863*pts.*

3. In 20 bushels, how many cubic inch measures? Ans. 43008.

TIME.

1. In 5 years, how many months, weeks, days, hours, minutes, and seconds?
Ans. 60*M.* 260*w.* 1826¼*d.* 43830*h.*
2629800*mi.* 157788000*sec.*

2. American independence was declared A. D. 1776, July 4th, how many seconds of time have elapsed from thence to January 1st, 1821?
Ans. 1404102600*sec.*

MOTION.

1. In 12 signs, how many degrees, minutes, seconds, thirds and fourths?
Ans. 360*deg.* 21600*mi.* 1296000*sec.*
77760000*thi.* 4665600000*four.*

REDUCTION ASCENDING.

1. In 500000 mills, how many cents, dimes, dollars, and eagles? Ans. 50000*c.* 5000*d.* 500*D.* 50*E.*

2. In 48000 farthings, how many pence, shillings, and pounds? Ans. 12000*d.* 1000*s.* 50£.

3. In 114488000 drams how many tons?
Ans. 19*T.* 19*C.* 1*qr.* 5*lb.* 14*oz.*

4. In 17280 grains Troy, how many pounds?
Ans. 3*lb.*

5. In 10000 grains Apothecaries weight, how many scruples, drams, ounces, and pounds?
Ans. 500*sc.* $166\frac{2}{3}$*dr.* $20\frac{5}{6}$*oz.* 1*lb.* $\frac{53}{72}$.

6. In 5890300 barleycorns, how many leagues?
Ans. 10*L.* 0*m.* 7*f.* 36*p.* 1*yd.* 2*ft.* 5*in.* 1*bc.*

7, In 10000000 inches, how many miles?
Ans. 157*m.* 6*fu.* 24*p.* 5*yds*; 2*ft.* 4*in.*

8. In 340498 nails, how many yards?
Ans. 21281*yds.* 0*qr.* 2*na.*

9. In 7843245 ells English, how many yards?
Ans. 9804056*yds.* 1*qr.*

10. In 1900 ells Flemish, how many yards?
Ans. 1425*yds.*

11. In 7648476 square perches, how many acres?
Ans. 47802*A.* 3*qr.* 36*P.*

12. In 100000000 square inches, how many acres?
Ans. 22*A.* 0*R.* 33*P.* $257\frac{1}{2}$*in.*

13. In 48746768 cubic inches, how many yards?
Ans. 1044*yds.* 21*ft.* 1616*in.*

14. In 34676769 cubic inch measures, how many gallons, wine measure?
Ans. 150115*gal.* 3*qts.* 1*pt.* $\frac{15}{231}$.

15. In 140049 cubic feet of stone work, how many perches, each $24\frac{3}{4}$ feet? Ans. $5658\frac{30}{55}$*per.*

16. In 4 walls, each 110 feet long, 2 feet thick, and 9 feet high, how many perches, of $24\frac{3}{4}$ feet?
Ans. 320*per.*

17. In 194746 pints of wine, how many tons?
Ans. 96*T.* 2*Hhd.* 25*gal.* 1*qt.*

18. In 190004 quarts, how many butts?
Ans. 376*butts*, 1*Hhd.* 62*gal.*

19. In 1000000 gallons of beer, how many hogsheads? Ans. 18518*Hhd.* 28*gal.*

20. In 700484 pints of wheat, how many bushels and quarts? Ans. 1368*q.* 1*bu.* 0*gal.* 2*qts.*

21. In 100000040 seconds of time, how many Julian years? Ans. 3*Y.* 2*M.* $\frac{1}{13}$*w.* 2*d.* 9*h.* 47*mi.* 20*sec.*

22. 1684000 seconds, how many circles?

Ans. 1*cir.* 107°, 46′, 40″.

23. In 18484000 dozens, how many common and great gross? Ans. 128361 *great gross*, 16*doz.*
and 1540333*com. gross*, 4*doz.*

24 If 8 Spanish dollars be an inch thick, how many would make a pile as high as the moon?

Ans. 12165120000*D.*

25. The reservoir belonging to the water works, near Philadelphia, being 317 feet long, 168 feet wide, and 9 feet deep, how many gallons of water does it contain? Ans. 3585442*gal.* $\frac{210}{231}$.

26. An eagle is about an inch broad, how many of them laid edge to edge, would reach from Harrisburg to Baltimore, a distance of 75 miles?

Ans. 4752000*E.*

27. How many quarter inch blocks will fill an inch box? Ans. 64 *blocks.*

28. In 1000 eagles, how many shares of dollars, dimes, cents, and mills, and of each an equal number?
Ans. 9000 of each sort, and 1000 mills over.

29. In 1600 pounds of beef, how many messes, each an equal number, of 1, 2, 3, 4, 5, and 6 pounds, can be made of the whole?

Ans. 76 of each, and 4*lb.* over.

30. Out of a ton of wine, how many jars of 1 gallon, ½ gallon, and quart, each an equal number, could be filled? 144 of each.

31. In £10000 sterling, how many English crowns, guineas, moidores, doubloons, and dollars, each an equal number?

RULE OF THREE.

1. The Rule of Three is so called, on account of three terms being always given, to find a fourth, which is the answer.

2. Some call it the rule of proportion; which teaches to compare lines with lines, numbers with numbers, magnitudes with magnitudes, &c. And by book 5, def. 8, Euclid, Proportion is called the similitude of ratios.

3. Three terms are necessary to constitute Proportion. Def. 9, book 5, Euclid.

4. Geometrical Proportion is that, which we use in all cases of the Rule of Three.

5. When the quantities or numbers are directly proportional, as 2, 4, 8, 16,

$$2 : 4 :: 8 : \frac{4\times8}{2}=16$$

Inversely, $$4 : 2 :: 16 : \frac{2\times16}{4}=8$$

Alternately, $$2 : 8 :: 4 : \frac{8\times4}{2}=16$$

Compoundedly, $$2 : 2+4 :: 8 : \frac{2+4\times8}{2}=8+16$$

Dividedly $$2 : 4-2 :: 8 : \frac{4-2\times8}{2}=16-8$$

Mixedly $$4+2 : 4-2 :: 16+8 : \frac{4-2\times16+8}{4+2}= 16-8$$

By multiplication $$2\times4 : 4\times4 :: 8 : \frac{4\times4\times8}{2\times4}=16$$

By division $$\tfrac{2}{2} : \tfrac{4}{2} :: 8 : \frac{\frac{4}{2}\times8}{\frac{2}{2}}=16^{*}$$

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* For note, see next page.

6. In each of the preceding sets of proportionals, the rectangles of means and extremes are equal; which must be so in all cases.

7. The Rule of Three Direct requires, that the three given terms be so placed, that the first may be the same way proportional to the second, that the third is to the fourth, or answer.

8. Homogeneous quantities only, admit of proper comparison; from which the following *rule* arises, viz.

RULE.

1. Write that number for the third term, which is of the same kind with the answer, or number sought.

2. Consider from the nature of the question, whether this third term is greater or less than the answer; and if greater, write the greater of the other two given numbers for the first term, and the less for the second; but if less, write the less for the first term, and the greater for the second; and the question is stated.

3. The reason for stating questions in the Rule of Three as above, is drawn from Def. 12, book 5, Eu-

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* Or, as $a, b, c, \frac{bc}{a}$, $a : b :: c : \frac{bc}{a}$ & $a \times \frac{bc}{a} = b \times c$.

Inversely, $b : a :: \frac{bc}{a} : c$ &c. &c.

Alternately, $a : c :: b : \frac{bc}{a}$

Compoundedly, $a : a+b :: c : c+\frac{bc}{a}$

Dividedly, $a : b-a :: c : \frac{bc}{a}-c$

Mixedly, $b \times a : b-a :: \frac{bc}{a}+c : \frac{bc}{a}-c$

By multiplication, $ab : b^2 :: c : \frac{bc}{a}$

By division, $\frac{a}{a} : \frac{b}{a} :: c : \frac{bc}{a}$

clid; where it is asserted, that of proportionals, the antecedent terms are homologous to one another, and also the consequents to one another.

4. Method of operation: Multiply the second and third terms together, and divide by the first.*

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* If it should be asked why we multiply the second and third terms of all statements in the Rule of Three Direct, and divide by the first, in order to find the answer; It is only necessary to observe the following illustration, viz. Let the numbers 2, 4, 8, and 16 be given,

then, 2 : 4 :: 8 : 16, which means that $\frac{4\times8}{2}=16$

And 4 : 2 :: 16 : 8, which means that $\frac{2\times16}{4}=8$

8 : 16 :: 2 : 4, which means that $\frac{16\times2}{8}=4$

And 16 : 8 :: 4 : 2, which means that $\frac{8\times4}{16}=2$

From the above it will be seen, that in every possible case, it is necessary that the second and third terms be multiplied, then divided by the first, in order to find the fourth proportional, which is the answer. If we take the letters a, b, c, and d, and say, as $a : b :: c : d$, $\frac{bc}{a}$ would be equal to d, and our proportion would now be $a : b :: c : \frac{bc}{a}=d$. If $\frac{bc}{a}=d$, then $bc=ad$, and $\frac{bc}{a}=d$.

Here all the values of a, b, c, and d naturally arise from the equation $\frac{bc}{a}=d$, and from the fact, that the rectangle of means and extremes must always be equal.

$\frac{ad}{b}=c$,
$\frac{ad}{c}=b$;
$\frac{bc}{d}=a$;

EXAMPLES.

1. If 1 yard of linen cost 75 cents, what will a piece of 30 yards come to?

yd. yds. cts. D. cts.
1 : 30 : : 75 : 22, 50 Ans.

2. If 1 yard of silk cost 8*s.* 4*d.* what will 40 yards come to?

yd. yds. s. d. £ *s. d.*
1 : 40 : : 8 4 : 16 13 4 Ans.

3. If 30 yards of linen cost 22 dollars 50 cents, what is that per yard?

yds. yd. D. cts. cts.
30 : 1 : : 22, 50 : 75 Ans.

4. If 40 yards of silk cost £16 13*s.* 4*d.* what is that per yard?

yds. yd. £ *s. d. s. d.*
40 : 1 : : 16 13 4 : 8 4 Ans.

5. If 1 ell of brown holland cost 37½ cents, what would 100 pieces, each 30 ells, come to?

E. P. E. cts. D.
1 : 100×30 : : 37½ : 1125 Ans.

6. What comes 100 pieces, each 30 ells, to, at 2*s.* 9½*d.* per ell?

E. P. E. s. d. £ *s.*
1 : 100×30 : : 2 9½ : 418 15 Ans.

7. If 100 pieces, each 30 ells, cost 1125 dollars, required the price of an ell.

P. E. E. D. cts.
100×30 : 1 : : 1125 : 37½ Ans.

8. If 100 pieces of brown holland, each 30 ells, cost £418 15*s.*, how much per ell?

P. E. E. £ *s. s. d.*
100×30 : 1 : : 418 15 : 2 9½ Ans.

9. If flour sell for 10 dollars 12½ cents per barrel, how much would 1000 barrels come to?

bl. bl. D. cts. D.
1 : 1000 : : 10, 12½ : 10125 Ans.

10 If flour sell for £3 15*s*. 11¼*d*. per barrel, what would 1000 barrels come to?

bl. *bl.* £ *s.* *d.* £ *s.* *d.*
1 : 1000 :: 3 15 11¼ : 3796 17 6 Ans.

11. If 1000 barrels of flour sold for $10125 how much per barrel?

bl. *bl.* *D.* *D.* *cts.*
1000 : 1 :: 10125 : 10, 12½ Ans.

12. Paid for 1000 barrels of flour, £3796 17*s*. 6*d*. what did it stand me per barrel?

bl. *bl.* £ *s.* *d.* £ *s.* *d.*
1000 : 1 :: 3796 17 6 : 3 15 11¼ Ans.

13. If 1 bushel of wheat be worth $1, 50*cts*. how much would 1500 bushels be worth?

bu. *bu.* *D. cts.* *D.*
1 : 1500 :: 1, 50 : 2250 Ans.

14. If 1 bushel cost 11*s*. 3*d*., how much would 1500 bushels come to?

bu. *bu.* *s.* *d.* £ *s.*
1 : 1500 :: 11 3 : 843 15 Ans.

15. If 1500 bushels cost £843 15*s*., what did it stand per bushel?

bu. *bu.* £ *s.* *s.* *d.*
1500 : 1 :: 843 15 : 11 3 Ans.

16. If a man spend one day with another $1, 75*cts*. how much would his expenses amount to in 20 years?

d. *d.* *Y.* *D. cts.* *D.* *cts.*
1 : 365¼ × 20 :: 1, 75 : 12783, 75 Ans.

17. If a poor man saves a quarter dollar per day, for 20 years, how much would he then be worth?

d. *d.* *Y.* *cts.* *D.* *cts.*
1 : 365¼ × 20 :: 25 : 1826, 25 Ans.

18. If a merchant in trade clears, one day with another, $5, 50*cts*. for 20 years, what is he worth?

d. *d.* *Y.* *D. cts.* *D.* *cts.*
1 : 365¼ × 20 :: 5, 50 : 40177, 50 Ans.

19. Suppose a man's estate would last him 20 years, and support a daily expense of $2, what stock did he commence his expenses with?

d. *d.* *Y.* *D.* *D.*
1 : 365¼ × 20 :: 2 : 14610 Ans.

20. If a man spends, one day with another, 6¼*cts.* for 49 years, required the amount of expenses.

d. *d.* *Y.* *cts.* *D. cts. m.*
1 : 365¼×49 :: 6¼ : 1118, 57 8 Ans.

21. If the hind wheel of a stage turn once on 18 feet of the road, how often will it turn in running from Harrisburg to Pittsburg, which is 186 miles?

ft. *m.* *ft.* *turn.* *turns.*
18 : 186×5280 :: 1 : 54560 Ans.

22. In common flush water, the Susquehanna, opposite to Harrisburg, moves at the rate of 5 miles an hour; at which time the river will average a mean depth of 6 feet. It is therefore required to determine the number of cubic feet of water which would pass the town in a year, provided the mean depth remain the same, and the breadth ¾ of a mile.

h. *d.* *breadth dep. dis. pr. h.* *cubic ft.* Ans.
1 : 365¼ :: 3960×6×5280×5 : 5498596224000

23. If one cubic foot of water weighs 64**lb.*, how many tons are contained in 5498596224000 cubic feet?

ft. *ft.* *lb.* *T.* *C. q. lb.*
1 : 5498596224000 :: 64 : 157102749257 2 3 12

24. If $100, in a year's time, give $6 interest, what amount of interest will $1000000 give in the same time?

D. *D.* *D. in.* *D. in.*
100 : 1000000 :: 6 : 60000 Ans.

25. A drove of 300 head of cattle crossed the bridge at Harrisburg, at 3 cents per head; required, the toll.

head. head. *cts.* *cts.* *D.*
1 : 300 :: 3 : 900=9 Ans.

26. A grazier charged 5*d.* per head for pasturing 350 cattle one night; required the bill of pasturage.

head. head. *d.* *£ s. d.*
1 : 350 :: 5 : 7 5 10 Ans.

---

* 62½*lb.* is the weight of pure water per cubic foot.

27. A farmer sold 40 bullocks, at £11 5*s.* each; required, their whole price.

bu. *bu.* £ s. £
1 : 40 : : 11 5 : 450 Ans.

28. For how many dollars should a check have been drawn, to pay the above sum?

£ £ *D. cts.* *D.*
1 : 450 : : 2, 66⅔ : 1200 Ans.

29. If 1½ bushels of oats cost 37½ cents, what will 400 bushels come to?

bu. *bu.* *cts.* *D.*
1½ : 400 : : 37½ : 100 Ans.

30. If a herring and a half cost three half pence, how many could I buy for 11*d.*?

d. *d.* *her.* *her.*
1½ : 11 : : 1½ : 11 Ans.

31. If 7 yards of cloth cost 49*s.*, what would 11 pieces, each 77 yards, come to?

yds. *pie.* *yds.* s. s. £ s.
7 : 11 × 77 : : 49 : 5929 = 296 9 Ans.

32. A piece of broad cloth, containing 38 yards, cost $190, how much per yard?

yds. *yds.* *D.* *D.*
38 : 1 : : 190 : 5 Ans.

33. If 20 pieces of linen, at 5*s.* per yard, cost £100, how many yards to a piece?

s. £ *yd.* *yds.* *yds.*
5 : 100 : : 1 : 400 = the whole, & $\frac{400}{20}$ = 20 pr. piece.

34. Bought 100 ells of brown holland for £25, and a piece of Irish linen the same length at 8*s.* 4*d.* per ell English; how many ells English was the length of the piece?

As the ell English is to the ell Flemish in the ratio of $\frac{3}{5}$

E.F. *E.E.*
5 : 3 : : 100 : 60 Ans.

35. What tax will an estate pay, which is valued at $14000, when 3½*cts.* are assesed on the dollar?

D. *D.* *cts.* *D.*
1 : 14000 : : 3½ : 490 Ans.

36. A merchant finds, after reckoning his expenses, losses, and other contingencies, that he cannot venture to lay more than 15 per cent on his merchandize; what should he sell an article for, which cost $35?

D. D. D. D. cts.
100 : 15 : : 35 : 5, 25 gain; therefore,
35+5, 25=$40, 25 Ans.

37. If the selling price of an article, at 15 per cent advance, was $40, 25; required, the prime cost.

D. D. D. D. cts. D. cts.
100+15 : 15 : : 40, 25 : 5, 25 gain; therefore,
40, 25—5, 25=$35 Ans.

38. What should cloth sell for, which cost $5, 50 per yard, when I add 20 per cent advance?

D. D. cts. D. D. cts.
100 : 5, 50 : : 20 : 1, 10=advance; therefore,
5, 50+$1, 10=$6, 60*cts.* Ans.

39. Bought an article for $100, which will bear 25 per cent profit; how should I sell it?

D. D. D. D.
100 : 100 : : 25 : 25 advance; therefore,
100+25=$125 Ans.

40. If a merchant retail $40000 worth of merchandize in a year, at 12½ per cent clear profit; required, the whole gain.

D. D. D. cts. D.
100 : 40000 : : 12, 50 : 5000 Ans.

41. If a merchant lose 3½ per cent, on $100000 worth of merchandize; required, the whole loss.

D. D. D. cts. D.
100 : 100000 : : 3, 50 : 3500 Ans.

42. An Iron-master blows a furnace constantly for two years, and casts three tons of pigs every eight hours, and these pigs sell for $30 per ton; what weight of pigs were in the whole, and what the price?

h. d. h. h. h. h. T. T.
1 : 365¼×2 : : 24 : 17532 & 8 : 17532 : : 3 : 6574½
T. T. D. D.
=whole weight; & 1 : 6574¼ : : 30 : 19737=price.

43. If a forge work 500 tons of bar iron in a year, how much would it sell for altogether, provided it would command $120 per ton?

T. *T.* *D.* *D.*
1 : 500 : : 120 : 60000 Ans.

44. If iron sell for a five penny bit per pound; required, the price of 19*C.* 3*q.* 17½*lb.*

lb. *C.* *q.* *lb.* *d.* £ *s.* *d.*
1 : 19 3 17½ : : $5\frac{1}{8}$: 47 12 $2\frac{3}{16}$ Ans.

45. If 1 pound of brown sugar retails for 2 cents more than it cost; required, the amount of profit on selling 10 hogsheads, each 10*C.* gross weight.

lb. *lb.* *C.* *lb.* *cts.* *D.*
1 : $112\times10\times10$: : 2 : 224 Ans.

46. A person failing in trade, gives up his property for the use of his creditors; how much is paid on the dollar, when the amount of debt was $14000, and property $10000?

D. *D.* *D.* *cts.*
14000 : 10000 : : 1 : $71\frac{3}{7}$ Ans.

47. The victualler of an army provided 40005*lbs.* of beef, to be distributed, 9*lbs.* to a mess of 6 men; required, the number of men.

lb. *lb.* *M.* *Men.*
$\frac{9}{6}=1\frac{1}{2}$ and $1\frac{1}{2}$: 40005 : : 1 : 26670 Ans.

48. The commander of a ship of war, laid in 136875*lb.* of bread for a six month's cruise; how many men had he on board, when each man had 1*lb.* per day?

days. *d.* *lb.* *lb.*
$182\frac{1}{2}$: 1 : : 136875 : 750 the daily ration of bread, consequently he had 750 men on board.

49. Suppose a crew of 150 men at sea, were put upon short allowance of water, each man a pint per day; how long would 2 casks, each containing 100 gallons, last them?

M. *M.* *gal.* *ca.* *pts.*
150 : 1 : : $100\times2\times8$: $10\frac{2}{3}$ pints each, and as a pint per day was the proper ration, it would last them $10\frac{2}{3}$ days.

50. If the ratio of 10 to 1 be 10; required, the ratio of 2 to 3.
10 : 1 = $\frac{10}{1}$ = 10, therefore, 2 : 3 = $\frac{2}{3}$, ratio required.

OF CONTRACTED OPERATIONS.

CASE 1.

If the first term be a multiple, or a part of the second, the third will be a like multiple, or a part of the fourth; and the answer will be found by multiplication or division.

EXAMPLES.

1. Bought 15 yards of linen for $9, what would 225 yards of the same cost?

yds. *yds.* *D.*
15)15 : 225 : : 9
D.
1 : 15 : : 9 : 135 Ans.

2. Bought 11 bushels of salt for $8; what would 121 bushels come to at the same rate?

bu. *bu.* *D.*
11)11 : 121 : : 8
D.
1 : 11 : : 8 : 88 Ans.

3. If 30 yards of broad cloth cost £39 12*s.* 6*d.*; how much should I have paid for 6 yards.

yds. *yds.* £ *s.* *d.*
6)30 : 6 : : 39 12 6
£ *s.* *d.*
5 : 1 : : 39 12 6 : 7 18 6 Ans.

4. If 30 bushels of wheat cost $40, how much would pay for 900 bushels?

bu. *bu.* *D.*
30)30 : 900 : : 40
D.
1 : 30 : : 40 : 1200 Ans.

4. If a coat pattern, of 2½ yards, superfine cloth, cost $30, what would a piece containing 30 yards come to?

yds. *yds.* *D.*
2½)2½ : 30 : : 30
——————— *D.*
1 : 12 : : 30 : 360 Ans.

It is evident from the above, that in many cases, operations in the Rule of Three may be abridged; for 7 : 49 : : 9 : 63 gives the same result only, which 1 : 7 : : 9 : 63 does; because 49 is the same multiple of 7, that 7 is of 1.

CASE 2.

If, when part of the first term is added to or subtracted from the first, the sum or remainder be equal to the second—a multiple of the second—or a part of the second; then accordingly a like part of the third term being added to, or subtracted from the third, the sum, or the remainder, will be equal to the fourth term—a like multiple of the fourth—or a like part of the fourth. Hence the answer is already found by case first of this rule.

EXAMPLES.

1. When the interest of a certain sum of money for 60 days is $85, 32*cts.*, what is the interest of the same sum for 75 days.

d. *d.* *D.* *cts.*
60 : 75 : : 85, 32
¼ of 60=15 21, 33=¼ of 85, 32
—— ——
75 $106, 65 Ans.

2. When 24 yards of linen cost £9 13*s.* 4*d.* how much must I pay for 21 yards of the same?

yds. *yds.* £ *s.* *d.*
24 : 21 : : 9 13 4 £ *s.* *d.*
⅛ of 24= 8 1 4 2=⅛ of 9 13 4
—— ——
21 £8 9 2

3. If 64 tons of iron sold for $7680, how much should 8 tons of the same sell for?

T. T. D.
64 : 8 :: 7680 D.
$\frac{1}{8}$ of 64 = 8 960 = $\frac{1}{8}$ of 7680
8)56(7 7)6720
$960 Ans.

That is, 64 minus $\frac{1}{8}$ of itself, is to 8, as 7680 minus $\frac{1}{8}$ of itself is to the answer; and because 8 measures 56 and 8 both, the proportion stands lastly

7 : 1 :: 6720 : 960 Ans.

CASE 3.

In other cases reduce the first and second terms to the same denomination, and consider them as pure numbers; then the product of the second and third terms, divided by the first, gives the answer in the denomination of the third term.

EXAMPLE.

If 1*C.* of sugar cost $17, what must I pay for 13*C.* 3*q.* 14*lb.*

C. *C.* *q.* *lb.* *D.*
1 : 13 3 14 :: 17
4 4
4 55
28 28
454
110
112 : 1554
16 : 222
8 : 111 :: 17 : 235, $87\frac{1}{2}$ Ans.

In like manner all cases of this kind may be done.

RULE OF THREE INVERSE.

1. Inverse, or inverted, Proportion, is generally defined by writers of Arithmetic to be such, that *more* requires *less*, or *less* requires *more; more* requires *less*, is when the third term is *greater* than the first, and the fourth *less* than the second.

2. *Less requires more*, is when the third term is less than the first, and the fourth greater than the second. These definitions may be illustrated by the following problems, viz. If in 10 days, 8 men can perform a piece of work, in what time could 40 men do the same?

$$\begin{array}{l}\quad\;\; d.\quad M.\quad\; M.\quad d.\\ \text{Solution[*]}\;\; 10 : 8 :: 40 : 2 \text{ or } \dfrac{10\times8}{40}=2 \text{ Ans.}\end{array}$$

Prob. 2. If 8 men can perform a piece of work in 10 days, how many men would be required to perform the same in 2 days?

$$\begin{array}{l}\quad\;\; M.\quad d.\quad\; d.\quad M.\\ \text{Solution}\;\; 8 : 10 :: 2 : 40 \text{ or } \dfrac{8\times10}{2}=40 \text{ Ans.}\end{array}$$

3. By the common methods given in the books, the above statements are correct; but are not so in fact.

4. In solution 1. we perceive that *more* requires *less;* for 10 days require 2 days; and in solution 2. *less* requires *more;* for 8 men require 40 men.

5. The three given numbers in each problem, as they stand, are by Prop. B. book 5, Euclid, in Inverted Proportion, because the third is to the second, as the first is to the fourth; that is,

$$\begin{array}{l} M.\quad M.\quad\; d.\quad d. \qquad\qquad d.\quad d.\quad\; M.\quad M.\\ 40 : 8 :: 10 : 2 \text{ and } 2 : 10 :: 8 : 40\end{array}$$

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[*] The above might answer for a stating, if the object of proportion was abandoned; but in all cases of the Rule of Three, legitimate proportion should be preserved, when the solutions are by no means rendered more difficult.

which is the correct method of stating them; for by Prop. 16, book 6, Euclid, the rectangles of means and extremes will be equal; for $40\times2=8\times10$, and $2\times40=10\times8$, which is not the case with the two first statements. By statements 1st and 2d, the answers are found by multiplying their first and second terms together, and dividing by the third.

6. By statements 3d and 4th, the answers are found by multiplying the second and third terms together, and dividing by the first; which is in fact multiplying the same terms together, and dividing by the same divisors as before: but the order of statements 1st and 2d, follow the order of the terms as they stand in the Problems, and accord with the opinions of most writers on Arithmetic, in which they are evidently mistaken.

7. In order to do away all abiguity about stating questions in Direct or Inverse Proportion, let the first rule given for stating questions in the Rule of Three Direct be strictly adhered to, and no possible difficulty can arise in either; homologous terms will then always come together in comparison, which is strictly mathematical.

EXAMPLES.

1. How much muslin that is ¾ of a yard wide will line 6¾ yards of stuff that is 1¾ yards wide?

qrs. yds. yds. yds. q.
¾ : 1¾ :: 6¾ : 15 3 Ans.

That is, as the breadth of the required piece, is to the breadth of the given piece; so is the length of the given piece, to that of the required piece.

2. How many yards of carpeting 2¾ feet wide, will cover a room 18 feet by 16?

yd. ft. ft. ft. ft. yd. yds. q. na.
$1=3\times2\frac{3}{4} : 18\times16 :: 1 : 34\ 3\ 2\frac{6}{11}$ Ans.

3. Suppose I lend my friend £350 for 5 months, he promising the like kindness; but when requested,

can spare but £125; how long may I keep it to balance my favour?

£ *£* *M.* *M.*
125 : 350 : : 5 : 14 Ans.

4. Suppose 450 soldiers are in a garrison, and their provisions are calculated to last but 5 months; how many must leave the garrison, that the same provisions may be sufficient for those who remain, 9 months?

m. *m.* *M.* *M.*
9 : 5 : : 450 : 250 the number who are to stay; therefore, 450—250=200 Ans.

5. If a man can perform a piece of work in 15 days of 12 hours long, in how many days of 10 hours long can he perform the same?

h. *h.* *d.* *d.*
10 : 12 : : 15 : 18 Ans.

6. If a piece of land 40 rods long and 4 broad make an acre, how wide must it be, if only 19 rods long, to make an acre?

R.l. *R.l.* *R.b.* *R.b.ft.in.*
19 : 40 : : 4 : 8 6 $11\frac{7}{19}$ Ans.

7. If when wheat is 6*s*. per bushel, the twopenny loaf weigh 10 ounces; what ought it to weigh, when wheat is 7*s*. 6*d*. per bushel?

s. *d.* *s.* *oz.* *oz.*
7 6 : 6 : : 10 : 8 Ans.

8. There is a cistern, having a pipe that will empty it in 6 hours; how many pipes of the same capacity will empty it in 20 mintes?

mi. *h.* *pi.* *pipes.*
20 : 6 : : 1 : 18 Ans.

9. If 4 men can thrash 20 bolls of wheat in 6 days, 12 hours long; how many will be required to do the same in an hour?

h. *d.* *h.* *M.* *M.*
1 : 6×12 : : 4 : 288 Ans.

10. If 9 persons can eat 100 loaves of bread in 15 days, allowing 3 meals per day; how many would be

required at the same rate of eating, to consume the 100 loaves at a meal?

meal meals per. per.
1 : 45 : : 9 : 405 Ans.

DOUBLE RULE OF THREE,

OR RULE OF FIVE.

1. In the Double Rule of Three, there are five terms given, to find a sixth, which is the answer; from this circumstance it is called the *Rule* of *Five*.

2. If we use two statings in the Rule of Three, to resolve questions which belong to the Rule of Five, we may then call it the Double Rule of Three.

3. If men, horses, &c. are given to perform work, journeys, &c. or money to produce interest, and time for those performances also given; the rule for stating questions, where five terms are given, is as follows, viz.

RULE.

1. Place the principal agent on the left, time next on its right, and the remaining name to the right of the last.

2. Place like names under each other, and a blank will stand open under that term which is of the same name with the answer.

3. If it so happens that the blank is found under the first or second term, the third and fourth are multiplied together for a divisor, and the remaining three for a dividend.

4. If the blank should fall under the third term, the first and second terms are multiplied together for a divisor, and the remaining three for a dividend.

RULE.

1. Place the numbers alternately, that is, the antecedents on the left hand, and the consequents on the right.

2. Let the last number stand on the left; multiply the left hand column continually for a dividend, and the right hand column for a divisor; divide, and the quotient will be the answer.

EXAMPLE.

1. Suppose 100 yards of America=100 yards of England, and 100 yards of England=50 canes of Toulouse, and 100 canes of Toulouse=160 ells of Geneva, and 100 ells of Geneva=200ells of Hamburg; how many yards of America are equal to 379 ells of Hambnrg?

Antecedents.		*Consequents.*
100 of America	=	100 of England.
100 of England	=	50 of Toulouse.
100 of Toulouse	=	160 of Geneva.
100 of Geneva	=	200 of Hamburg.
379 of Hamburg.		

$$\frac{100\times100\times100\times100\times379}{100\times50\times160\times200}=\frac{379\times5}{8}=236\tfrac{7}{8}\ yds.$$

The above may be cancelled, by striking equals from both sides; for instance, if any number on the left be equal to one on the right, both may be stricken out; if a number on the left be a multiple of its opposite on the right, strike out both, but reserve to such left hand number its quotient divided by the right hand number. When all are thus cancelled, work with the remainders, and you obtain the same answer as though you worked with the whole.

NOTE. In Practice the cancelled numbers are marked with a dash drawn across them.

CASE 2.

1. If 82 bushels at London=27 muddles at Amsterdam, and 27 muddles at Amsterdam=19 settiers at Paris, 38 settiers at Paris=65 verteels at Antwerp, 65 verteels at Antwerp=104 hanegas at Cadiz, 52 hanegas at Cadiz=216 alquires at Lisbon; how many alquires at Lisbon, are equal to 410 bushels at London?

82= 27
27= 19
38= 65
65=104
52=216 $\frac{27\times19\times65\times104\times216\times410}{82\times27\times38\times65\times52}=216$
410
$\times5=1080$ alquires a. Lisbon, equal 410 bushels at London.

Note. In case 2, the last number is placed on the right, on account of the nature of the demand.

FRACTIONS,

USUALLY CALLED VULGAR.

1. A fraction is defined to be part of a quantity. (Mathematical Correspondent, page 117.)

2. *Quantity*, is magnitude and its various parts, expressed by means of some known measure, considered as an infinitely divisible unit.

3. The parts of quantity are expressed by means of numbers, placed in a particular manner, thus, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, &c.

4. When a quantity is supposed to be divided into four equal parts, $\frac{1}{4}$ expresses the fourth part of it; $\frac{1}{2}$, half of it; and $\frac{3}{4}$, three fourths of it.

5. The figure above the line, is called the numerator, because it points out the number of the parts of the quantity; the figure below the line, is *called* the denominator, because it designates the number of parts the quantity is divided into.

6. Such expressions as $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, &c. viz. all expressions which have the greater number below the line, are called proper fractions.

7. All expressions as, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, &c. which have the greater number above the line, are called improper fractions.

8. Expressions, such as $\frac{1}{4}$ of $\frac{3}{4}$, $\frac{3}{4}$ of $\frac{5}{8}$, &c. are called compound fractions.

9. Expressions, such as $2\frac{1}{2}$, $3\frac{3}{4}$, $7\frac{5}{9}$, &c. are called mixed fractions.

10. The above, comprises the principal forms in use, in common Arithmetic.

REDUCTION OF FRACTIONAL EXPRESSIONS.

By Reduction, we change the given forms of fractions, to others, more simple or more complex.

CASE 1.

1. To reduce fractional expressions to their lowest or simplest terms, the *rule* is; divide the greater term by the less, the less by the remainder, the last divisor still again, until nought remain.

2. by that which divides and leaves nothing, divide both terms of the given fraction, and the thing is done.

EXAMPLES.

1. Reduce $\frac{336}{896}$ to its lowest terms.

```
Operation.  336)896(2
                672
                ———
                224)336(1
                    224
                    ———
                    112)224(2
                        224
```

We find that 112 divides without leaving a remainder.

Therefore, $112)\frac{336}{896}(\frac{3}{8}$ Ans.

2. Reduce $\frac{2832}{12848}$ to its lowest terms. Ans. $\frac{177}{803}$.

3. Reduce $\frac{144}{560}$ to its lowest terms. Ans. $\frac{9}{35}$.

4. Reduce $\frac{174326}{394896}$ to its lowest terms.
Ans. $\frac{87163}{197448}$.

NOTE. If both terms of the given fraction end with cyphers, strike equals from both, and proceed with the remaining parts as before.

5. Reduce $\frac{19000}{27000}$ to its lowest terms.
Operation. $\frac{19|000}{27|000}=\frac{19}{27}$ Ans.

6. Reduce $\frac{16000}{24600}$ to its lowest terms. Ans. $\frac{80}{123}$.

CASE 2.

1. This case requires a method of reducing a compound fraction to a single one.

2. To do this, multiply together all the numerators, for a new numerator, and all the denominators, for a new denominator; reduce this new fraction to its lowest terms, and the thing is done.

EXAMPLES.

1. Reduce $\frac{4}{9}$ of $\frac{5}{7}$ of $\frac{9}{10}$ to a single fraction.
Operation. $\frac{4}{9}\times\frac{5}{7}\times\frac{9}{10}=\frac{180}{630}=\frac{2}{7}$ Ans.

2. Reduce $\frac{2}{9}$ of $\frac{3}{4}$ of 4 to a single fraction.
Ans. $\frac{2}{3}$.

3. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a single fraction.
Ans. $\frac{2}{5}$.

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4. Reduce $\frac{1}{4}$ of $\frac{2}{5}$ of $\frac{3}{7}$ of 19 to a single fraction. Ans. $\frac{57}{70}$.

CASE 3.

1. This case requires a method of reducing whole and mixed numbers to improper fractions.

2. If the whole number has no given denominator, a unit written underneath must be the denominator.

3. If the whole number has a given denominator, multiply such number by it, and the product will be the numerator to the given denominator.

4. If a fraction be annexed to the whole number, multiply the whole number by the denominator of the fraction, and to the product add the numerator for a new one, which place over the denominator of the given fraction, and the thing is done.

EXAMPLES.

1. Reduce 12, 27, and 176, to fractions whose denominators shall be 1. Ans. $\frac{12}{1}$, $\frac{27}{1}$, and $\frac{176}{1}$.

2. Reduce 49 to a fraction, whose denominator shall be 13. Ans. $\frac{637}{13}$.

3. Reduce $4\frac{2}{7}$ to an improper fraction. Ans. $\frac{30}{7}$.

4. Reduce $145\frac{17}{19}$ to an improper fraction. Ans. $\frac{2772}{19}$.

CASE 4.

1. This case requires a method of reducing an improper fraction to its equivalent, or proper terms.

2. To do this, we must divide the numerator by the denominator, the quotient will be the whole part; if a remainder be left, place it over the denominator, and it will be the fractional part.

EXAMPLES.

1. Reduce $\frac{11}{3}$ to its proper terms. Ans. $3\frac{2}{3}$.

Operation. 3)11($3\frac{2}{3}$ Ans.
9
—
2

2. Reduce $\frac{1149}{17}$ to its proper terms. Ans. $67\frac{10}{17}$.
3. Reduce $\frac{1638}{19}$ to its proper terms. Ans. $86\frac{4}{19}$.
4. Reduce $\frac{7434}{175}$ to its proper terms. Ans. $42\frac{12}{25}$.

CASE 5.

1. This case requires a method of reducing fractions of different denominators, to fractions of the same value, that shall have a common denominator.

2. To do this, we must multiply each numerator by all the denominators but its own, for a new numerator, and all the denominators for a common denominator, and the thing is done.

EXAMPLES.

1. Reduce $\frac{3}{4}$, $\frac{11}{12}$ and $\frac{7}{10}$ to a common denominator.

Operation. $3 \times 12 \times 10 = 360 =$ 1st new numerator.
$11 \times 4 \times 10 = 440 =$ 2d.
$7 \times 4 \times 12 = 336 =$ 3d.
$4 \times 12 \times 10 = 480 =$ common denominator. Therefore the required fractions are, $\frac{360}{480}$, $\frac{440}{480}$, and $\frac{336}{480}$.

The same may be done as follows, viz.

$$\begin{array}{r|rrr} 2 & 4, & 12, & 10 \\ \hline 2 & 2 & 6 & 5 \\ \hline & 1 & 3 & 5 \end{array}$$

$$\begin{array}{r} 5 \\ 3 \\ \hline 15 \\ 2 \\ \hline 30 \\ 2 \\ \hline 60 \end{array}$$

$\frac{60 \times 3}{4} = 45 =$ 1st numerator.
$\frac{60 \times 11}{12} = 55 =$ 2d.
$\frac{60 \times 7}{10} = 42 =$ 3d.

60 common denominator.

Therefore, $\frac{45}{60}$, $\frac{55}{60}$, and $\frac{42}{60}$ are fractions of the same value with the former.

2. Reduce $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of $\frac{7}{8}$ to a common denominator. Ans. $\frac{960}{2880}$, $\frac{720}{2880}$, $\frac{576}{2880}$, $\frac{420}{2880}$.

3. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{8}{9}$ to a common denominator.

Ans. $\frac{3240}{6480}$, $\frac{2160}{6480}$, $\frac{1620}{6480}$, $\frac{1296}{6480}$, $\frac{1080}{6480}$, $\frac{5760}{6480}$.

4. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, and $\frac{11}{12}$ to a common denominator.

Ans. $\frac{20160}{30240}$, $\frac{22680}{30240}$, $\frac{24192}{30240}$, $\frac{25200}{30240}$, $\frac{26920}{30240}$, $\frac{27720}{30240}$.

CASE 6.

1. This case requires a method of reducing fractions of one denomination to another, retaining the same value.

2. To do this, when any parts of a shilling, pound, &c. are given, to be reduced to the fraction of a guinea, half eagle, &c. reduce both to the same name, place the lesser over the greater, and you have the fraction required.

3. If any parts of a pound, guinea, &c. be given, to be reduced to the fraction of a shilling, penny, &c. proceed as before, and the thing is done.

EXAMPLES.

1. Reduce $\frac{3}{4}$ of a shilling to the fraction of an English guinea.

Operation. $\frac{3}{4} \times 12 = \frac{36}{4} = 9d.$

1 guinea $\times 21 \times 12 = 252d.$ therefore, $\frac{9}{252} = \frac{1}{28}$, the fraction required.

2. Reduce $\frac{1}{820}$ of a £ to the fraction of a penny.

Ans. $\frac{12}{31}d.$

3. Reduce $\frac{3}{4}$ of a dollar to the fraction of an eagle.

Ans. $\frac{3}{40}E.$

4. Reduce $\frac{7}{8}$ of a cent to the fraction of a dollar.

Ans. $\frac{7}{800}D.$

5. Reduce $\frac{9}{10}$ of a guinea, Pennsylvania currency, to the fraction of an eagle. Ans. $\frac{4203}{10000}$.

6. Reduce $\frac{5}{6}$ of a grain to the fraction of a pound Troy. Ans. $\frac{1}{6912}$lb.

7. Reduce $\frac{7}{8}$ of a pound Avoirdupoise to the fraction of a dram. Ans. $\frac{224}{1}$ *dr.*

8. Reduce $\frac{7}{12}$ of a pound to the fraction of 1 *C.*
Ans. $\frac{1}{192}$ *C.*

9. Reduce $\frac{1}{144}$ of a league to the fraction of a pole.
Ans. $\frac{20}{3}$ *p.*

10. Reduce $\frac{11}{75}$ of a yard to the fraction of a nail.
Ans. $\frac{176}{75}$ *na.*

11. Reduce $\frac{11}{13}$ of a gallon of wine to the fraction of a hogshead. Ans. $\frac{11}{819}$ *Hhd.*

12. Reduce $\frac{7}{9}$ of a barrel of ale to the fraction of a pint.
Ans. $\frac{1792}{9}$,$=199\frac{1}{9}$ pints.

13. Reduce $\frac{13}{15}$ of a chaldron to the fraction of a bushel.
Ans $\frac{156}{5}=31\frac{1}{5}$ *bu.*

14. Reduce $\frac{1}{39}$ of a week to the fraction of a second.
Ans. $\frac{201600}{13}$ *sec.*

CASE 7.

This case requires a method of finding the proper quantity or value of a fraction in money, weights, measures, &c.

RULE.

Multiply the numerator of the given fraction by the parts of the money, weight, or measure unit, to which it belongs; divide that product by the denominator when the first part is multiplied by; if any thing remain, multiply the next part, divide as before, and so proceed to the last, and the thing is done.

EXAMPLES.

1. Reduce the $\frac{7}{13}$ of an eagle to its proper value.

```
        7
       10
       —
   13)70(5D.
       65
       —
        5
       10
       —
    13)50(3d.
        39
        —
        11
        10
        —
    13)110(8c.
        104
        ——
          6
         10
         —
     13)60(4m.
         52
         —
          8
```

2. Reduce $\frac{7}{8}$ of a dollar to its proper value.
Ans. 87½cts.

3. Reduce $\frac{5}{12}$ of an English guinea to its proper value.
Ans. 8s. 9d.

4. Reduce $\frac{9}{10}$ of a moidore, Pennsylvania currency, to its proper value.
Ans. £2 0s. 6d.

5. Reduce $\frac{7}{8}$ of a pound Troy to its proper value.
Ans. 10oz. 10dwt.

6. Reduce $\frac{11}{15}$ of a ton to its proper value.
Ans. 14C. 2q. 18lb. 10⅔oz.

7. Reduce $\frac{9}{10}$ of a league to its proper quantity.
Ans. 2m. 5f. 24p.

8. Reduce $\frac{19}{20}$ of an ell English to its proper quantity.
Ans. 1*yd*. 0*q*. 3*na*.

9. Reduce $\frac{34}{57}$ of a hogshead of wine to its proper quantity.
Ans. 37*gal*. $2\frac{6}{19}$*qts*.

10. Reduce $\frac{13}{17}$ of an acre to its proper quantity.
Ans. 3*R*. $2\frac{6}{17}$*p*.

CASE 8.

This case requires a method of reducing money, weights, and measures, to fractions.

RULE.

Reduce the given quantity to the lowest name mentioned, for a numerator, under which put those contained in an unit of the integer for a denominator; reduce this fraction to its lowest terms, and the thing is done.

EXAMPLES.

1. Reduce 6*s*. 8*d*. to the fraction of a pound.
6s. 8*d*.=80*d*.
20 =240 and $\frac{80}{240}=\frac{1}{3}$ Ans.

2. Reduce $7 9*d*. 8*c*. to the fraction of an eagle.
Ans. $\frac{399}{500}$ of an eagle.

3. Reduce 3*C*. 3*q*. 20*lb*. to the fraction of a ton.
Ans. $\frac{11}{56}$ of a ton.

4. Reduce 3 *R*. 36*p*. to the fraction of an acre.
Ans. $\frac{39}{40}$ of an acre.

CASE 9.

This case requires a method by which fractions may be reduced from one denomination to another of the same value, having the numerator of the required fraction given.

RULE.

As the numerator of the given fraction
Is to the denominator;
So is the numerator of the intended fraction,
To its denominator.

EXAMPLES.

1. Reduce $\frac{3}{4}$ to a fraction of the same value, whose numerator shall be 15.

Operation. 3 : 4 :: 15 : 20 Ans. and $\frac{15}{20}$ the required fraction.

2. Reduce $\frac{9}{10}$ to a fraction of the same value, whose numerator shall be 90. Ans. $\frac{90}{100}$.

CASE 10.

This case requires a method of reducing fractions from one denomination to another of the same value, having the denominator of the required fraction given.

RULE.

As the denominator of the given fraction,
Is to its numerator;
So is the denominator of the intended fraction,
To its numerator.

EXAMPLES.

1. Reduce $\frac{3}{4}$ to a fraction of the same value, whose denominator shall be 20.

Operation. 4 : 3 :: 20 : 15 and $\frac{15}{20}$ Ans.

2. Reduce $\frac{11}{12}$ to a fraction of the same value, whose denominator shall be 60. Ans. 55.

CASE 11.

This case requires a method of reducing a mixed fraction, of the following form, $\frac{42\frac{7}{8}}{48}$ to a simple one.

RULE.

Multiply each term of the principal fraction by the denominator of that annexed, for the like term of the simple fraction, adding the annexed numerator to the product of the term to which it belongs, and the thing is done.

EXAMPLES.

1. Reduce $\frac{42\frac{7}{8}}{49}$ to a simple fraction.

$$\text{Operation. } \begin{array}{l} 42\times 8+7=343 \\ 49\times 8 \quad\;\; =392 \end{array} \quad \frac{343}{392}=\frac{7}{8} \text{ Ans.}$$

2. Reduce $\frac{52\frac{4}{5}}{60}$ to a simple fraction. Ans. $\frac{22}{25}$.

ADDITION OF FRACTIONS.

RULE.

1. Prepare the given fractions, by reducing them to a common denominator, (if necessary,) add the numerators together.

2. Divide their sum by the common denominator, and the thing is done.

EXAMPLES.

1. Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$ together.

$$\left.\begin{array}{l} 1\times 4\times 5\times 6=120 \\ 3\times 2\times 5\times 6=180 \\ 4\times 2\times 4\times 6=192 \\ 5\times 2\times 4\times 5=200 \end{array}\right\}\text{numerators.}$$

$$2\times 4\times 5\times 6=240 \text{ common denominators.}$$

$$\frac{020+180+192+200}{240}=2\tfrac{53}{60} \text{ sum required.}$$

2. Add $\frac{7}{8}$, $\frac{9}{10}$, $\frac{11}{12}$, and $\frac{13}{14}$ together. Ans. $3\frac{521}{841}$.

3. Add $\frac{3}{5}$ of $\frac{4}{9}$, 6, $7\frac{3}{7}$, $11\frac{42}{43}$, and $\frac{14}{3}$ together.
Ans. $30\frac{34}{387}$.

4. Add $\frac{3}{4}$ of a cent, $\frac{7}{8}$ of a dollar, and $\frac{9}{10}$ of an eagle together. Ans. \$9, 88$\frac{1}{4}$.

5. Add $\frac{4}{5}$ of a ton, $\frac{9}{10}$ of a hundred, $\frac{7}{8}$ of a pound, and $\frac{5}{8}$ of an ounce together. Ans. 16*C.* 3*q.* 17*lb.* 11*oz.*$\frac{19}{34}$.

6. Add $\frac{7}{8}$ of a £, $\frac{9}{10}$ of a shilling, and $\frac{7}{8}$ of a penny together. Ans. 18*s.* 5*d.*$\frac{27}{40}$.

7. A person purchased of A, B, C, and D, respectively, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{19}$ of a ship valued at $15000; what did the four shares cost him in all? Ans. 14955\frac{55}{57}$.

8. Add $\frac{1}{2}$ of a moidore, 5 doubloons, 5 English guineas, $\frac{9}{10}$ of a French crown, $\frac{7}{8}$ of a dollar, (reduced to Pennsylvania currency,) together. Ans. $102, 86¼.

9. Add $\frac{1}{2}$ of $\frac{3}{4}$ of 2½ dollars, $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of 3¾ eagles, and $\frac{8}{9}$ of $\frac{9}{10}$ of $\frac{11}{12}$ of 75 cents, together.

Ans. 2*E.* 3*D.* 3*d.* 6¼*cts.*

10. Add 7 years, $\frac{9}{16}$ weeks, $\frac{11}{12}$ months, $\frac{13}{14}$ days, 49 hours, and $\frac{37}{75}$ minutes together.

Ans. 7*Y.* 1*M.* $\frac{2}{3}$*w.* 2*d.* 2*h.* 18*mi.*$\frac{26}{35}$.

SUBTRACTION OF FRACTIONS.

RULE.

Prepare the given fractions as in Addition; take the difference of the numerators, under which subscribe the common denominator, and the thing is done.

EXAMPLES.

1. Take $\frac{1}{12}$ from $\frac{8}{9}$.*

Operation. $\frac{8}{9}-\frac{1}{12}=\frac{8\times12-1\times9}{9\times12}=\frac{87}{108}$ Ans.

2. From 34¾ eagles, take 19 $\frac{7}{8}$ dollars.

Ans. $327, 62½*cts.*

3. From 19½† moidores, take 20$\frac{7}{8}$ guineas, each being reckoned sterling currency. Ans. £4 18s. 1½*d.*

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[*] When we say, take $\frac{1}{12}$ from $\frac{8}{9}$, we mean that the difference between the $\frac{8}{9}$ parts of a magnitude, considered as an unit, and the $\frac{1}{12}$ parts of the same is a homogeneous quantity, because the minuend and subtrahend are pure fractional expressions, without any definite relation to any thing but such magnitude.

[†] When I say, take a dollar from an eagle, I speak ambiguously; for I mean, that the difference between the sums of like money contained in those pieces are to be taken.

4. From $\frac{9}{10}$ of a ton, take $\frac{11}{12}$ of $\frac{13}{18}$ of $2\frac{1}{4}$ *C.*
Ans. 15*C.* 3*q.* $7\frac{4}{9}$*lb.*

5. From $14\frac{3}{8}$ yards, take $\frac{1}{2}$ of $\frac{9}{10}$ of $\frac{4}{3}$ of $3\frac{1}{2}$ ells English.
Ans. $11\frac{3}{4}$*yds.*

6. From $\frac{4}{8}$ of a hogshead of spirits, take $\frac{3}{4}$ of $\frac{11}{13}$ of $1\frac{1}{2}$ barrels. Ans. 20*gal.* 1*qt.* 1*pt.* $\frac{41}{130}$.

7. From $103\frac{7}{8}$ acres, take $\frac{3}{7}$ of $\frac{14}{19}$ of $70\frac{1}{9}$ acres.
Ans. 85*A.* 3*R.* 3*p.* $\frac{66}{399}$.

MULTIPLICATION OF FRACTIONS.

RULE.

1. Reduce compound fractions to simple ones, and mixed numbers to improper fractions (when necessary.)

2. Multiply the numerators together for a new numerator, and the denominators, for a new denominator, and the thing is done.

EXAMPLES.

1. Multiply $\frac{3}{4}$ and $\frac{5}{8}$ together.

Operation. $\left.\begin{array}{l} 3\times 5=15 \text{ numerator} \\ 4\times 8=32 \text{ denominator} \end{array}\right\}\frac{15}{32}$ Ans.

2. Multiply $\frac{3}{4}$ of $\frac{5}{8}$ by $\frac{9}{10}$ of $\frac{11}{12}$.
Ans. $\frac{1485}{3840}=\frac{99}{256}$.

3. Multiply $9\frac{7}{10}$ by $5\frac{3}{4}$. Ans. $55\frac{31}{40}$.

4. Multiply $33\frac{1}{3}$ by $\frac{3}{4}$ of $\frac{4}{5}$ of $19\frac{1}{2}$. Ans. 390.

5. Multiply $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $11\frac{1}{2}$, by $\frac{6}{7}$ of $3\frac{1}{2}$.
Ans. $6\frac{9}{10}$.

6. Multiply $19\frac{11}{12}$, by $\frac{13}{14}$ of $19\frac{11}{12}$.
Ans. $368\frac{685}{2016}$.

7. Bought $100\frac{1}{2}$ barrels of flour, for $3\frac{7}{8}$ dollars per barrel; what came they to? Ans. \$389, $43\frac{3}{4}$.

8. Bought $37\frac{3}{4}$ yards of linen, at $\frac{5}{8}$ of a dollar per yard; required the whole cost. Ans. \$23, $59\frac{3}{8}$.

9. Bought $90\frac{3}{4}$ acres of land, for $75\frac{5}{8}$ dollars per acre; required, the price of the whole.
Ans. \$6862, $96\frac{7}{8}$.

10. Bought $25\frac{3}{5}C.$ of iron, for $5\frac{7}{8}$ dollars per $C.$ required, the price. Ans. $151, $28\frac{1}{8}$.

DIVISION OF FRACTIONS.

RULE.

1. Prepare the fractions as in multiplication; multiply the denominator of the divisor into the numerator of the dividend, for a new numerator.

2. Multiply the numerator of the divisor into the denominator of the dividend, for a new denominator.

3. Let this new fraction be reduced to its lowest terms, or to an improper one, and the thing is done.

Note. By inverting the terms of the divisor, and working as in multiplication, the same thing is obtained.

EXAMPLES.

1. Divide $\frac{9}{10}$ by $\frac{3}{5}$.

Operation by the rule. $\frac{3}{5})\frac{9}{10}(=\frac{9\times5}{10\times3}=\frac{45}{30}=1\frac{1}{2}$ Ans.

By the above note. $\frac{5}{3}\times\frac{9}{10}=\frac{45}{30}=1\frac{1}{2}$ Ans.

2. Divide $20\frac{3}{4}$ by $30\frac{3}{4}$. Ans. $\frac{83}{123}$.

3. Divide $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $9\frac{1}{2}$ by $11\frac{3}{8}$.
Ans. $\frac{76}{455}$.

4. Divide $\frac{11}{12}$ of 3, by $\frac{7}{8}$ of 6. Ans. $\frac{11}{21}$.

5. Divide $\frac{63}{84}$ of 10, by $\frac{21}{23}$ of 12. Ans. $\frac{115}{168}$.

6. Divide $100\frac{1}{2}$, by $\frac{99}{100}$ of 9. Ans. $11\frac{83}{297}$.

7. Divide $750\frac{7}{8}$ dollars amongst 19 persons.
Ans. $68, $26\frac{3}{22}$ share of each.

8. Divide $\frac{3}{4}$ of $800\frac{1}{2}$ dollars amongst 11 persons.
Ans. $54, $57\frac{21}{22}$ share of each.

SINGLE RULE OF THREE IN FRACTIONS.

RULE.

1. State the given terms the same way as first taught by rule, page 79.

2. Prepare the given terms in a proper manner, and work as there directed, and the thing is done.

EXAMPLES.

1. If $\frac{3}{4}$ of a yard of cloth cost \$$2\frac{1}{2}$, what would be the price of $30\frac{3}{4}$ yards?

$$\begin{array}{lcccccccc} & yd. & & yds. & & D. & & D. \\ \text{Operation.} & \frac{3}{4} & : & 30\frac{3}{4} & :: & 2\frac{1}{2} & : & 102\frac{1}{2} \\ & & & 4 & & 2 & & \\ \hline & & & \frac{123}{4} & & \frac{5}{2} & & \end{array}$$

$$\tfrac{3}{4})\tfrac{123}{4}\times\tfrac{5}{2}(=\tfrac{4}{3}\times\tfrac{123}{4}\times\tfrac{5}{2}=\tfrac{2460}{24}=102\tfrac{1}{2}.$$

2. If $\frac{3}{5}$ of a yard cost $\frac{7}{15}$ of a £, what cost $\frac{3}{14}$ *yds.* Ans. 3*s.* 4*d.*

3. If $\frac{3}{4}$ of $\frac{5}{8}$ of a ship cost \$10000, what is the remainder worth? Ans. \$$11333\frac{1}{3}$.

4. If \$20 be divided amongst four persons, in the ratio of $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$; required, the share of each.

$$\text{Ans.}\begin{cases} 7\frac{1}{57}. \\ 5\frac{5}{19}. \\ 4\frac{4}{19}. \\ 3\frac{29}{57}. \end{cases}$$

5. If the $\frac{1}{3}$ of 9 be 11; required, the $\frac{1}{2}$ of 6, and $\frac{1}{4}$ of 7. Ans. 11 and $6\frac{5}{12}$.

6. If the ratio of 1 to 3 be $\frac{1}{3}$; required, the ratio of 3 to $4\frac{1}{3}$. Ans. $\frac{9}{13}$.

7. If $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of a ship, be worth $\frac{8}{9}$ of $\frac{7}{8}$ of $\frac{5}{7}$ of her cargo, valued at \$4000, what is her whole value? Ans. \$$8888\frac{8}{9}$.

8. If a silver dollar weighs 17*dwt.* $1\frac{3}{4}$*gr.* Troy, how many tons Avoirdupoise will \$1000000 weigh? Ans. 26*T.* 2*C.* 2*q.* $15\frac{5}{7}$*lb.*

9. In one ton of standard gold, how many eagles, each 11*dwt.* $4\frac{3}{4}$*gr*?

Ans. $64011\frac{1003}{1075}$=$$640119\frac{7}{215}$.

10. If 175 ounces Troy, be equal to 192 Avoirdupoise, how many tons gross weight, would 5000000 of dimes make, each 1*dwt.* $16\frac{9}{10}$*gr.*?

Ans. 13*T.* 0*C.* 3*q.* $10\frac{2}{7}$*lb.*

11. If a French guinea in Federal currency, be $4, $62\frac{26}{27}$, how many of them would be worth $9000?

Ans. 1944 French guineas.

12. If the $\frac{1}{3}$ of a farm less $\frac{1}{8}$, be worth $\frac{3}{4}+\frac{1}{9}$ of $8000, at $$20\frac{3}{4}$ per acre; required, the content of the farm.

Ans. $1991\frac{241}{249}$ acres.

13. A spent $\frac{19}{63}$ of his estate, the residue he disposed of as follows, viz. To his eldest son $\frac{19}{63}$ of the remainder, to his second son $\frac{19}{63}$ of what was left, and the remainder to his residuary legatee, which exactly amounted to $1000; required, the value of the estate.

Ans. $$2935\frac{3995}{10648}$.

QUESTIONS FOR EXERCISE.

1. How many stones of $1\frac{3}{4}$ feet long, $\frac{2}{3}$ foot broad, and $\frac{3}{4}$ foot thick, are equal to 50 stones of $3\frac{1}{3}$ feet long, $2\frac{1}{2}$ feet broad, and $1\frac{1}{5}$ foot thick.

$1\frac{3}{4}\times\frac{2}{3}\times\frac{3}{4} : 3\frac{1}{3}\times2\frac{1}{2}\times1\frac{1}{5} :: 50 : 571\frac{3}{7}$ Ans.

2. How much will 2 bags of wool come to, No. 1 $=94\frac{3}{16}$ stone, No. $2=305\frac{7}{9}$ stone, at 10*s.* $6\frac{2}{3}$*d.* per stone; but $4\frac{2}{3}$ stone of No. 2, are but $2\frac{1}{4}$ stone of No. 1?

st. *st.* *st.* *st.*

$4\frac{2}{3} : 2\frac{1}{4} :: 305\frac{7}{9} : 147\frac{3}{7}$ of No. 1.

And $147\frac{3}{7}+94\frac{3}{16}=241\frac{69}{112}$, therefore,

$1 : 241\frac{69}{112} :: 10\ 6\frac{2}{3} : 147\ 10\ 4\frac{59}{84}$ Ans.

3. A father devised $\frac{34}{83}$ of his estate to one of his sons, $\frac{34}{83}$ of the remainder to another, and the surplus to his relict for life; the sons' legacies were found to be $1285 different; what money did he leave the widow the use of?

Whole estate$=\frac{83}{83}$ and $\frac{83}{83}-\frac{34}{83}=\frac{49}{83}$.

Then, $\frac{34}{83}+\frac{49}{83}=\frac{1666}{6889}$=youngest son's part.

$\frac{34}{83}=\frac{2822}{6889}=$eldest son's part.

Therefore, $\frac{2822}{6889}-\frac{1666}{6889}=\frac{1156}{6889}=\$1285$, and $\frac{2822}{6889}+\frac{1666}{6889}$ $=\frac{4488}{6889}$ both sons' shares.

So that $\frac{6889}{6889}-\frac{4488}{6889}=\frac{2401}{6889}=$widow's part.

Omitting the common denominators, we have

1156 : 2401 :: 1285 : 2668 $93\frac{281}{578}$ Ans.

4. A man dying, gave to his eldest son $\frac{2}{3}$ of $\frac{1}{4}$ of his estate; to his second, $\frac{1}{5}$ of $\frac{1}{2}$; and when they counted their portions, the one had $240 more than the other; the remainder was given to the wife and younger children; how much had each?

$\frac{2}{3}$ of $\frac{1}{4}=\frac{2}{12}=\frac{1}{6}=$eldest son's share;

And $\frac{1}{5}$ of $\frac{1}{2}=\frac{1}{10}$, second son's share.

And $\frac{1}{6}-\frac{1}{10}=\frac{1}{15}=240$, therefore,

1 : 15 :: 240 : 3600=whole estate.

And $\frac{3600}{6}=\$600$, eldest son's share;

$\frac{3600}{10}=\$360$, second son's share;

And $3600-\overline{600+360}=2640$, part left for widow and younger children.

5. A, in a scuffle, seized on $\frac{2}{3}$ of a parcel of sugar plums; B caught $\frac{3}{8}$ of them out of his hands; and C laid hold on $\frac{3}{10}$ more; D ran off with all A had left, except a $\frac{1}{7}$, which E afterwards took slyly for himself; then A and C jointly set upon B, who, in the conflict, dropped half he had, which were equally picked up by D and E, who said belonged to them per due. B then kicked down C's hat, and to work they all went anew for what it contained; of which A got $\frac{1}{4}$, B $\frac{1}{2}$, D $\frac{3}{7}$, and C and E equal shares of what was left of that stock; D then struck $\frac{3}{4}$ of what A and B last acquired, out of their hands; they with difficulty recovered $\frac{5}{8}$ of them in equal shares again; but the other three took off $\frac{1}{5}$ a piece of the same. Upon this they called a truce, and agreed, that the $\frac{1}{3}$ of the whole left by A at first, be equally divided among them. How much of the prize, after this distribution, remained with each of the competitors?

First $\frac{3}{8}$ of $\frac{2}{3}=\frac{1}{4}$B's $\Big\}$ first acquisition, their sum$=\frac{9}{20}$.
And $\frac{3}{10}$ of $\frac{2}{3}=\frac{1}{5}$C's $\Big\}$

Then $\frac{2}{3}-\frac{9}{20}=\frac{13}{60}=\frac{91}{420}$ left.

$\frac{1}{7}$ of $\frac{13}{60}=\frac{13}{420}=$E's first acquisition.

Also, $\frac{91}{420}-\frac{13}{420}=\frac{78}{420}$ D's. Thus ended the first heat.

Again, $\frac{1}{2}$ of $\frac{1}{4}=\frac{1}{8}$ B's
Retained $\frac{1}{5}$ C's
And $\frac{13}{70}+\frac{1}{16}=\frac{139}{560}$ D's
Also $\frac{13}{420}+\frac{1}{16}=\frac{157}{1680}$ E's
} part at the end of the second scuffle.

Proceeding, $\frac{1}{4}$ of $\frac{1}{5}=\frac{1}{20}$ A's.
$\frac{1}{5}$ of $\frac{1}{3}+\frac{1}{8}=\frac{23}{120}$ B's.
$\frac{2}{7}$ of $\frac{1}{5}+\frac{139}{560}=\frac{171}{560}$ D's.
Then $\frac{1}{20}+\frac{1}{15}+\frac{2}{35}=\frac{73}{420}$.
$\frac{1}{5}-\frac{73}{420}=\frac{11}{420}$, and $\frac{1}{2}$ of $\frac{11}{420}=\frac{11}{840}$ C's
$\frac{157}{1680}+\frac{11}{840}=\frac{179}{1680}$ E's
} part of the third smuss.

Further $\frac{1}{20}+\frac{1}{15}=\frac{7}{60}$, and $\frac{3}{4}$ of $\frac{7}{60}=\frac{7}{80}$, lost by A & B.
Then $\frac{5}{16}$ of $\frac{7}{80}+\frac{1}{4}$ of $\frac{1}{20}=\frac{51}{1280}$ A's.
Also $\frac{5}{16}$ of $\frac{7}{80}+\frac{1}{4}$ of $\frac{1}{15}$ of $\frac{1}{8}=\frac{649}{3840}$ B's
$\frac{1}{8}$ of $\frac{7}{80}+\frac{11}{840}=\frac{323}{13440}$ E's
} Part after the last smuss.
$\frac{1}{8}$ of $\frac{7}{80}+\frac{171}{840}=\frac{1417}{4480}$ D's
$\frac{1}{8}$ of $\frac{7}{80}+\frac{179}{1680}=\frac{1579}{13440}$ E's
} Part after the last smuss.

Then $\frac{51}{1280}+\frac{1}{15}=\frac{2863}{26880}$ A's
And $\frac{649}{3840}+\frac{1}{15}=\frac{6335}{26880}$ B's
Also $\frac{323}{13440}+\frac{1}{15}=\frac{2438}{26880}$ C's
$\frac{1417}{4480}+\frac{1}{15}=\frac{10294}{26880}$ D's
$\frac{1579}{13440}+\frac{1}{15}=\frac{4950}{26880}$ E's
} Share carried off at the last.

So that if the number of sugar plums were 26880,

A got 2863
B 6335
C 2438
D 10294
E 4950
} = 26880, the sum.

DECIMAL FRACTIONS.

1. A decimal fraction is the quotient resulting from dividing the numerator of a vulgar fraction by its denominator, and is called so many tenths, hundredths, thousandths, &c. of any magnitude or other

thing, considered as an unit, according as such quotient consists of one, two, three, &c. placed to the right of the decimal point.

2. Every decimal fraction is reducible to a vulgar fraction equivalent thereto.

3. Decimal fractions have the same ratio that whole numbers have.

4. The following forms show how decimals are written, viz.

.1	This is read, one tenth.
.02	This, two hundredths.
.003	This, three thousandths.
.12	This, twelve hundredths.
.123	This, one hundred & twenty-three thousandths.

Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Tens of thousandths.	Hundreds of thousandths.	Millionths.	Tens of millionths.
7	6	5	4	3	2	1.	1	2	3	4	5	6	7

From the above scheme, it evidently appears, that whole numbers and decimals have the same ratio.

REDUCTION OF DECIMALS.

CASE 1.

This case requires a method of reducing any given vulgar fraction to a decimal, equivalent thereto.

RULE.

Divide the numerator by the denominator, and the thing is done.

EXAMPLES.

1. Reduce the fractions $\frac{4}{5}$ and $\frac{7}{8}$ to equivalent decimals.

Operation. 5)4.0(.8 Ans.=eight tenths.

8)7.0(.875 Ans. = eight hundred and seventy-five thousandths; for $\frac{875}{1000}=\frac{7}{8}$=the vulgar fraction from which .875 was derived.

2. Reduce $\frac{9}{10}$ to a decimal. Ans. .9
3. Reduce $\frac{11}{20}$ to a decimal. Ans. .55
4. Reduce $\frac{15}{40}$ to a decimal. Ans. .375
5. Reduce $\frac{40}{320}$ to a decimal. Ans. .125
6. Reduce $\frac{12}{240}$ to a decimal. Ans. .05

CASE 2.

This case requires a method of reducing such vulgar fractions to decimals, as have no even divisors.

RULE.

Proceed, as in case 1; but when you arrive at six places of decimals stop; the fraction will then be near enough the truth for all practical purposes. The above named fractions are called circulates.

EXAMPLES.

1. Reduce $\frac{1}{3}$ to a decimal. Ans. .333333+
2. Reduce $\frac{2}{3}$ to a decimal. .666666+
3. Reduce $\frac{9}{11}$ to a decimal. .818181+
4. Reduce $\frac{11}{17}$ to a decimal. .647058+

CASE 3.

To reduce fractions of divers denominations to decimals.

RULE.

1. Add a cypher or cyphers, as before, to the est name given.

2. Divide by that number, which makes one o highest, and the thing is done.

EXAMPLES.

1. Reduce 15 cents to the decimal of a dollar
   Operation. 100)150(.15 Ans.
2. Reduce 11*s.* to the decimal of a £.
   Ans. £
3. Reduce 7*d.* to the decimal of a shilling.
   Ans. .58333
4. Reduce 10*oz.* Troy, to the decimal of a *lb.*
   Ans. .83333
5. Reduce 14*oz.* Avoirdupoise, to the decim a *lb.* Ans. .
6. Reduce 6*lb.* to the decimal of a *C.*
   Ans. .05357
7. Reduce 3*na.* to the decimal of a yard.
   Ans. .1
8. Reduce 7*yds.* to the decimal of a mile.
   Ans. 00397
9. Reduce 3*C.* 3*q.* 7*lb.* to the decimal of a ton
   Ans. .190
10. Reduce 1 cubic foot to the decimal of a y
    Ans. .03703

CASE 4.

To find the value of any decimal, in known p

RULE.

1. Multiply by the highest name mentioned; off as many places from the right of the produc are in the given decimal.

2. Multiply the figures thus cut off, by the next highest name; cut off again, and so proceed to the last, and the thing is done.

EXAMPLES.

1. What is the value of .875 of a £?

Operation. .875
20
17.500
12
6.000 Ans. 17*s.* 6*d.*

2. What is the value of .375 of a ton?
Ans. 7*C.* 2*q.*

3. What is the value of .666666 of 3*q.* 24*lb.*?
Ans. 71.999999+.

4. What is the value of .87 of a mile?
Ans. 6*f.* 38*p.* 2*yds.* 0*ft.* 7*in.* 2.

ADDITION OF DECIMALS.

Addition of decimals is performed precisely as that of whole numbers; but the quantities must be properly ranked first, by placing units under units, *tens* under tens, tenths under tenths, hundredths under hundredths, &c.

EXAMPLES.

Tons.	*Hhds.*	*Yds.*	*Miles.*
164.0346	3.75	3.005	.06
39.17	1.195	4.2	.004
33.14675	2.875	5.1745	.753
18.875	1.0875	1.75	.6025
13.0875	3.140675	9.110335	.90465
9.0625	1.000425	1.999425	.67432

SUBTRACTION OF DECIMALS.

Subtraction of decimals is performed precisely as that of whole numbers; but care must be taken to place units under units, tens under tens, &c. as in addition.

EXAMPLES.

Yds.	*Feet.*	*Miles.*	*Hhds.*
45.04319	8.194301	100.5	13.75
10.95326	2.009735	.9125	1.875

MULTIPLICATION OF DECIMALS.

Multiplication of decimals is performed precisely as that of whole numbers; but there must be reserved in the product as many decimal places, as are contained in both factors taken together; and the thing is done.

EXAMPLES.

Multiply 6.75
By 8.04 Or, 6.75 × 8.04 = 64.27 Ans.

2700
5400

54.2700 Here four decimal places are reserved, because each of the factors contain two.

Multiply	74.0475	.04375	.12345
By	.7546	.00388	.54321

Note. When two decimal factors are such, that their product will not make as many places as are contained in both factors, the defect must be made up by cyphers, placed on the left of the significant figures.

EXAMPLE.

Multiply .1234
By .1234

Operation .1234 × .1234 = .01522756.

The factors themselves make but 1522756, when multiplied, only seven places; which is supplied, by placing a cypher to their left, as above.

To multiply decimals, so that the product shall consist of a determinate number of places to the right of the decimal point,

RULE.

1. Invert the order of the multiplier, thus, viz. suppose it was 123, or 1.23, it must be put 321, or 32.1, and place it under the multiplicand so that the figure which stands immediately to the right of the decimal point, may be directly under the first, second, third, &c. decimal place of the multiplicand, according as you wish to reserve one, two, three, &c. decimal places.

2. Multiply as in common cases, with the following cautions, viz. When the units figure of the multiplier stands under the units figure of the multiplicand, multiply by it as in common cases; but when multiplying by the second, third, &c. figures of the multiplier, only multiply that figure of the multiplicand, which stands to its right, reserving the product in your mind, carrying to the product of that which

stands immediately above it, 1 from 5 to 15*, 2 from 15 to 25, 3 from 25 to 35, &c. placing the first figure of each product in a perpendicular line, under the right hand figure of the first product, and the thing is done.

EXAMPLE.

Multiply 12.345 by 12.345 and reserve in the product two and three decimal places.

12.345	12.345
543.21	543.21
12345	12345
2469 ·	24690
370 · ·	3704
49 · · ·	494
6 · · · ·	61
152.39 first ans'r.	152.399 second ans'r.

These two examples are sufficient to illustrate the rule, which is very convenient in many cases; a few decimal places being sufficiently exact for practice.

NOTE. The above rule is not always strictly true, sometimes giving a little too much, and other times something defective; but the difference from truth does not materially affect any result.

DIVISION OF DECIMALS.

Division of decimals is performed precisely as that of whole numbers; but care must be taken to point off as many decimal places to the right of the quotient, as the divisor wants of those in the dividend, and the thing is done.

---

* The reason for carrying 1 from 5 to 15, 2 from 15 to 25, &c. readily appears in example 1; such allowance being necessary to restore what is lost by the reversion of the several products.

EXAMPLES.

1. Divide 3.74896 by .347642.

.347642)3.74896(10.783967 +
3 47642
———
2725400
2433494
———
2919060
2781136
———
1379240
1042926
———
3363140
3128778
———
2343620
2085852
———
2577680
2433494
———
144186 remainder.

Here it is observable, that six places must be cut off to the right of the quotient, because the dividend contains twelve places of decimals, which is the number contained in both divisor and quotient exactly.

2. Divide 47.64÷64.74. Ans. .735866+
3. Divide .34743÷75. .0046324
4. Divide 12345.÷54321. .227260+

If we desired to divide any number, as 7393. by

10.	7393.	Ans. 739.3
100.		73.93
1000.		7.393
10000.		.7393
100000.		.07393

It is only necessary to cut off as many decimal places to the right hand of such number, as there are cyphers in the divisor, and the thing is done.

CONTRACTED DIVISION OF DECIMALS.

RULE.

When you perform the operation to find the first figure of the quotient, let the remainder stand for a new dividend, without increasing it by bringing a figure or figures from the given dividend; do so in each case, pointing off one figure from the right of the divisor for each new dividend, observing at each operation, to carry the increase of the figures cut off, in multiplication, and the quotient will be had nearly true.

EXAMPLE.

Divide 14169.206623851 by 384.672158

```
384.672158)14169.206623851(36.8345+ Ans.
     ,,,,,  11540 16474
            ----------
             262904188
             230803294
             ---------
              32100894
              30773772
              --------
               1327122
               1154016
               -------
                173106
                153868
                ------
                 19238
                 19233
                 -----
                     5
```

EXAMPLES,

SHEWING COMMON PRACTICAL USES OF DECIMALS.

1. What will a piece of cloth, containing 50 yards, come to at $3.75 per yard? Ans. $187.50.

2. What will 750 shingles come to at $12. per thousand? Ans. Ans. $9.

3. What will 15.450 feet of boards come to at $15.50 per thousand? Ans. $239.47½

4. What will 350 gallons of whiskey come to at $.75 per gallon? Ans. $262.50

5. What will 200 bushels of wheat come to at $1.60 per bushel? Ans. $320.

6. What does the interest of $1575 come to in one year, at 5, 5½, and 6 per cent? Ans. $259.87½

Operation. $$\left.\begin{array}{l} 1575 \times .05 = 78.75 \\ 1575 \times .055 = 86.62.5 \\ 1575 \times .06 = 94.50 \end{array}\right\} \text{Ans.}$$

NOTE. Decimal Arithmetic should be carefully studied by every student that would be a man of business. It is applicable to almost all kinds of computations. Our money unit being in a decuple, or tenfold ratio, makes it indispensably necessary to be acquainted with decimals. A thorough knowledge of vulgar fractions should be previously studied, being the radix from whence decimals flow.

RULE OF THREE IN DECIMALS.

The Rule of Three in decimals is subject to the same rules as that of whole numbers.

EXAMPLES.

1. If 3.25*lb.* of flour cost 9.5*cts*, what would a barrel of flour come to, the weight being 196.*lb.*

lb. *lb.* *cts.*
3.25 : 196 :: 9.5
9.5

980
1764

3.25)1862.0($5.72.9 + Ans.
1625

2370
2275

950
650

3000
2925

75

2. If .75 of a ton of iron cost $80.50, what is the price of 9.5 tons?

T. *T.* *D.* *D.*
.75 : 9.5 :: 80.50 : 1019.66⅔ Ans.

3. If .875 of a pipe of wine cost $104.25, what would a ton come to?

Pi. *T.* *D.* *D.*
.875 : 1 :: 104.25 : 238.28⁴⁄₇ Ans.

4. .375 of a farm contained exactly 100 acres, how many acres would 4.75 such tracts contain?
Ans. 1266⅔ acres.

5. .0625 of a cargo of tea came to $100; required, the amount of the whole. Ans. $1600.

6. If a man spend .125 of his estate in 2.5 years, how long will the whole last him? Ans. 20 years.

CIRCULATING DECIMALS.

A circulating decimal is, in other words, an indeterminate or unlimited decimal, representing only an approximate value for the vulgar fraction from which it arises; and is of such a nature, that it will sometimes repeat the same figure continually, or repeat every second, third, &c. in the same manner; from which arises the doctrine of simple, compound, and mixed repetends; similar, dissimilar, and conterminous. But as these quaint distinctions serve more for curiosity than real use, I shall content myself with barely giving some of the most prominent definitions.

The following fraction represents

A single repetend,	$.\dot{3}33333$, &c.*
A compound repetend,	$.\dot{1}2\dot{3}123$, &c.
A mixed single repetend,	$.08\dot{3}333$, &c.
A mixed compound repetend,	$.73\dot{5}1\dot{4}5145$, &c.

Similar repetends which begin at the same place, are said to be conterminous.

.333 &c. and .111 &c. are similar repetends.
.333 &c. and .083 &c. are dissimilar.

Note. It is generally time to stop, when any decimal whatever, of any shape, runs to six places; being sufficiently accurate for all practical purposes.

Note 2. In calculations, where circulates are concerned, the first repeating figure is frequently marked with a dash drawn across it.

---

* It may appear strange to a student, to hear it roundly maintained, that it is possible to shew the precise sum of a progression which consists of an infinite number of terms, when no one in existence can have a distinct idea of the magnitude of such a number. $\frac{1}{3}$, reduced to a decimal, will run on for ever, .333, &c. constantly approaching its ultimate, without being able to come up. Hence a faint notion of infinity.

DOUBLE RULE OF THREE.

EXAMPLES.

1. If \$100, in .75 year, gain \$4.5 interest, what principal will give \$9. in .25 year? Ans. \$600.

2. If 40 men, in 19.5 days, earn \$60.5; how many men, in 20 days, will earn \$155$\frac{5}{39}$? Ans. 100 men.

3. If 12 men mowed 15.5 acres of grass, in 15.5 hours, how many acres may be mowed by 100 men, in 2.25 hours? Ans. 18.75 acres.

EXTENSIVE ADDITIONS.

Such as frequently occur in bank, and other large monied establishments.

RULE.

Add each column by itself, and place the respective aggregates in such order, that units, tens, hundreds, &c. may stand in their proper places; lastly, add those aggregates together, and you have the whole sum.

EXAMPLE.

$7464.16
176.44
3000.00
199.99
5000.75
800.75
900.64
1800.76
19875.75
160.75
87.45
16.75
8000.00
750.75
75.68
1870.50
174.60

NOTE. It will be much more convenient to use this method of addition for large accounts; because, if an error be made in the summing of any particular column, it may be detected without interfering, or a dependance on any of the other columns; and it makes no difference whether we begin at the right hand, or the left, or the middle.

72 = sum of first column.
90 = sum of second column.
46 = sum of third column.
70 = sum of fourth column.
56 = sum of fifth column.
34 = sum of sixth column.
1 = sum of seventh column.

$50355, 72 = sum total.

The use of the preceding method of addition in bank accounts.

Amounts of notes.	*Discounts for 60 days.*	*Proceeds of notes.*
$3000.	$32.	$2968.
1800.	19.20	1780.80
450.	4.80	445.20
1106.	11.80	1094.20
500.	5.33	494.67
600.	6.40	593.60
750.	8.00	742.
800.	8.53	791.47
900.	9.60	890.40
1000.	10.67	989.33
1200.	12.80	1187.20
1500.	16.00	1484.
100.	1.07	98.93
150.	1.60	148.40
200.	2.13	197.87
250.	2.67	247.33
6	.30	30
20.	63.	61.
61..	66..	77..
8...	8...	117...
		69....
14306. notes.	152.60 discounts	6.....
		14153.40 proceeds.
		152.60 discounts.
Am't of proceeds & discounts		14306.00

The nature of the above account is such, that the sum of the left hand rank, viz. the amount of notes, must be equal to the sums, both of discounts and proceeds, exactly; else the additions are not right, for they prove each other. If any errors arise, they may be readily discovered, without adding the whole together, or without a dependance on each other, as in the common mode. The error, or errors, may, perhaps, be discovered in going over but one rank of the figures, which may be mostly known by inspection.

PRACTICE.

Practice is a commodious method of casting up accounts, principally by short division and addition.

RULE.

1. Prepare the given price, if under a £, $, &c. by dividing it into aliquot parts; divide the given quantity by those parts, add the quotients together, and the thing is done.

2. If the given price be above a £, $, &c. multiply by the whole part, divide by the aliquot parts of the remainder; add as before, and the thing is done.

PRACTICE TABLE.

Aliquot parts of 1*d*.

$\frac{1}{2}$	=	$\frac{1}{2}$
$\frac{1}{4}$	=	$\frac{1}{4}$

Aliquot parts of 1£.

s.	*d*.		
10		=	$\frac{1}{2}$
6	8	=	$\frac{1}{3}$
5		=	$\frac{1}{4}$
4		=	$\frac{1}{5}$
3	4	=	$\frac{1}{6}$
2	6	=	$\frac{1}{8}$
2		=	$\frac{1}{10}$
1	8	=	$\frac{1}{12}$
1		=	$\frac{1}{20}$
	6	=	$\frac{1}{40}$
	8	=	$\frac{1}{30}$
	4	=	$\frac{1}{60}$
	3	=	$\frac{1}{80}$
	2	=	$\frac{1}{120}$
	1	=	$\frac{1}{240}$

Aliquot parts of 1*s*.

6*d*. is	=	$\frac{1}{2}$
4	=	$\frac{1}{3}$
3	=	$\frac{1}{4}$
2	=	$\frac{1}{6}$
1½	=	$\frac{1}{8}$
1	=	$\frac{1}{12}$

Aliquot parts of a $.

50*cts*.	=	$\frac{1}{2}$
25	=	$\frac{1}{4}$
20	=	$\frac{1}{5}$
12½	=	$\frac{1}{8}$
10	=	$\frac{1}{10}$
6¼	=	$\frac{1}{16}$
33⅓	=	$\frac{1}{3}$
37½	=	$\frac{3}{8}$
62½	=	$\frac{5}{8}$
75	=	$\frac{3}{4}$
87½	=	$\frac{7}{8}$

Aliquot parts of 1*C.*

2*qrs.* = $\frac{1}{2}$
1 = $\frac{1}{4}$
14*lb.* = $\frac{1}{8}$
16 = $\frac{1}{7}$
8 = $\frac{1}{14}$
7 = $\frac{1}{16}$

Aliquot parts of ½*C.*

28*lb.* = $\frac{1}{2}$
14 = $\frac{1}{4}$
7 = $\frac{1}{8}$
8 = $\frac{1}{7}$
4 = $\frac{1}{14}$
2 = $\frac{1}{28}$
1 = $\frac{1}{56}$

EXAMPLES.

1. 3400*lb.* at ¼*d.* per *lb.*

¼	¼	3400
	12	850
	2|0	7|0 10
		£3 10 10 Ans

	Answer.
2. 4074 at ½*d.*	£8 9 9
3. 9477 at ¾.	29 12 3¾
4. 17040 at 1¼.	88 15 0
5. 34783 at 1½*cts.*	$521, 74½
6. 14740 at 1¾.	257, 95*
7. 7444 at 2.	148, 88
8. 4798 at 2¼.	107, 95½
9. 1994 at 2½.	49, 85
10. 1964 at 2¾.	54, 01
11. 1739 at 3.	52, 17
12. 994 at 3¼.	32, 30½
13. 1876 at 3½.	65, 66
14. 1775 at 3¾*cts.*	$66, 56¼
15. 9874 at 4.	394, 96
16. 887 at 4¼.	37, 69¾
17. 7467 at 4½.	336, 01½
18. 964 at 4¾.	45, 79
19. 1740 at 5.	87, 00
20. 944 at 5¼.	49, 56
21. 1888 at 5½.	103, 84
22. 799 at 5¾.	45, 94¼
23. 18740 at 6.	1124, 40
24. 4746 at 6¼.	296, 62½
25. 7443 at 6½.	483, 79½
26. 7432 at 6¾.	501, 66
27. 6749 at 7.	472, 43
28. 789 at 7¼.	57, 20¼
29. 674 at 7½.	50, 55
30. 4546 at 7¾.	352, 31½
31. 8742 at 8.	699, 36
32. 9400 at 8¼.	775, 50
33. 7775 at 8½.	660, 87½
34. 10409 at 8¾.	910, 78¾

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* These examples will all answer for English money; and in my opinion, it would be well to cause the student to resolve them for both English and Federal money. For the examples done above for Federal money, the quantities may be multiplied by the whole parts of the price, and parts taken for the fractional; the sum then divided by 100, or by having two places to the right cut off for cents, the answer is had.

	Answer.
35. 9874 at 9*cts*.	$888, 66
36. 7432 at 9¼.	687, 46
37. 1149 at 9½.	109, 15½
38. 13470 at 9¾.	1313, 32½
39. 8747 at 10.	874, 70
40. 14090 at 10¼.	1444, 22½
41. 1714 at 10½.	179, 97
42. 987 at 10¾*cts*.	$106, 10¼
43. 674 at 11.	74, 14
44. 1664 at 11¼.	187, 20
45. 17400 at 11½.	2001, 00
46. 1999 at 11¾*d*.	£97 17 4¼
47. 20000 at 12.	500 00 0

1. 5047 at 6¼*cts*.

6¼=$\frac{1}{16}$ | 5047

$315, 43¾ Ans.

	Answer.
2. 79480 at 10*cts*.	$7948, 00
3. 19404 at 12½.	2426, 00
4. 74603 at 18¾.	13988, 06¼
5. 14404 at 25.	3601, 00
6. 17467 at 31¼.	5458, 43¾
7. 16740 at 37½.	6277, 50
8. 12404 at 43¾.	5426, 75
9. 17404 at 50*cts*.	$8702, 00
10. 74004 at 56¼.	41627, 25
11. 9474 at 62½.	5921, 25
12. 19474 at 68¾.	13388, 37½
13. 17404 at 75.	13053, 00
14. 9467 at 81¼.	7691, 93¾
15. 19474 at 87½.	17039, 75
16. 19467 at 93¾.	18250, 31¼
17. 14040 at 1.6¼.	14917, 50
18. 7440 at 1.12½.	8370, 00
19. 6094 at 1.25.	7617, 50

If the price per yard, *lb.* &c. be any even number of shillings, the amount is had by multiplying the number of things by half the given price, doubling the first product for shillings.*

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* The reason why we multiply by half the number of shillings in the given price, and double the first product for shillings, is easily discovered in the following example, viz. Let 1762 things be given at 4 shillings each, to find their price; 1762×4=7048, which divided by 20, gives £352 8*s*. the price of the whole: but if we had just multiplied by 2, half the given price, doubling the first product for shillings, we have obtained the same thing without dividing by 20 at all, thus:

1762
2
————
£352 8 Answer as before;

which is nothing but multiplying by $\frac{2}{10}=\frac{4}{20}$; for $1762\times\frac{4}{20}=1762\times\frac{2}{10}=1762\times\frac{1}{5}=$ £352 8 as above, the second equality being that upon which the rule is founded.

Given, 17*C.* 3*qrs.* 21*lb.* of iron at £2 9*s.* 6*d.* per *Cwt.*

s. d.	*C.*	*q.*	*lb.*
6 8 = ⅓	17	3	21
	2	9	6
	34		
1 8 = ¼	5	13	4
10 = ½	1	8	4
		14	2
2 = ⅕ }		2	10
2 = ⅕ }		2	10
	2	6	4¾
	£44	7	10½ Answer.

q.	£	*s.*	*d.*
2 = ½	2	9	6
1 = ½	1	4	9
14 = ½		12	4½
7 = ½		6	2¼
		3	1
	£2	6	4¾ Price of 3*qrs.* 21*lb.*

From the number of examples already exhibited, all other cases in which the rule of Practice is necessary, can be easily resolved.

* * *

TARE AND TRET.

Tare is an allowance made by the seller to the buyer, for the weight of the case, cask, &c. in which the goods are packed; reckoned at so much per cask, or at so much per cent, according to the nature of the goods.

Tret is an allowance made on some articles, on account of waste, and is deducted from the *Suttle.*

Suttle is the weight which remains after Tare is deducted.

Nett weight is the quantity to be settled for after all deductions are made.

TABLE OF ALLOWANCES,

ESTABLISHED BY ACT OF CONGRESS.

FOR DRAFT.

On any quantity of 100 or 112*lb.*		1*lb.*
Above 100*lb.*	and under 200*lb.*	2
200	300	3
300	1000	4
1000	1800	7
1800 or upwards,		9

TARES.

On every chest of bohea tea,	70*lb.*
Half do.	36
Quarter do.	20
A chest of hyson or green tea, of 70*lb.* and upwards,	20
A box of other tea, between 50 and 70*lb.*	18
If 80	20
From 80 and upwards,	22

On all other teas, according to their actual weight.

	per cent.
On coffee in bags,	2
bales,	3
casks,	12
On sugar in casks,	12
boxes,	15
bags or mats,	5
On cocoa in casks,	10
bags,	1
On pimento in casks,	16
bags,	3
On cheese in baskets,	10

	per cent.
On cheese in boxes,	20
On chocolate in boxes,	10
On cotton in bales,	2
seroons,	6
On indigo in barrels,	12
other casks,	15
seroons,	10
On pepper in casks,	12
bales,	5
bags,	2
On shot in casks	3

On all other goods, according to the invoice, as actual weight.

CASE 1.

INVOICE TARE.

RULE.

Add the *gross* weight into one sum, and the tare into another; subtract the tare from the gross; the remainder is the *nett* weight.

EXAMPLE.

Required the value of 4 casks of merchandize, at 68¾*cts.* per *lb.* number and weight as follows:

	C.	*q.*	*lb.*		*lb.*
No. 1.	4	1	10	tare	36
2.	3	3	2		29
3.	4	0	19		32
4.	4	0	0		35
	16	1	3		132=whole tare.

4

65

4

260

7

1823

132

1691 neat at 68¾*cts.* comes to

$1162, 56¼. Ans.

CASE 2.

When *tret* is deducted from the *gross* weight, add the *gross* as before; then carry the *tret* under the *tare*, and subtract the sum of both from the *gross*.

EXAMPLE.

What is the value of 2 *Hhds.* of merchandize, allowing 1*lb.* *tret* for every *Cwt.* out of the *gross*, at $3, 56 per *Cwt.*; weight as follows?

	C.	*q.*	*lb.*		*C.*	*q.*	*lb.*	
No. 1.	10	1	11	tare	0	3	20	
No. 2.	11	0	17		0	3	14	
							21½	tret.
Gross,	21	2	0	at 1*lb.*				
	1	3	27½		1	3	27½	sum of tare and tret.

50=½ | 19 2 0½
3 56
57
5=1/10 | 9 50
1=1/5 | 95
lb. 19
Price of 56½ 1 79½
=69, 43½ Ans.

qrs.
2=½ | 3 56
1 78
½*lb.*= 1½
1, 79½ price of 56½*lb.*

CASE 3.

When *tret* is allowed after the *tare* is deducted, which is generally at 4*lb.* on every 104*lb.* or 1/26 part.

EXAMPLE.

What cost 5 packs of wool, gross 21*C.* 0*q.* 8*lb.* *tare* 28*lb.* per pack, *tret* 8*lb.* for every 3*Cwt.* at $13, 60 per *Cwt.*?

C. *q.* *lb.*
5 packs = 21 0 8 gross.
5 × 28 = 1 1 0 tare.
8*lb.* = $\frac{1}{42}$ | 19 3 8 suttle.
0 1 $24\frac{6}{7}$
50 = $\frac{1}{2}$ | 19 1 $11\frac{1}{7}$ neat.
13 60
247
10 = $\frac{1}{5}$ | 9 50
1 90
4 $73\frac{1}{2}$ price of 1*q.* 11*lb.*
$263 $13\frac{1}{2}$ Ans.

1*q.* = $\frac{1}{4}$ | 13 60
7*lb.* = $\frac{1}{4}$ | 3 40
$3\frac{1}{2}$ = $\frac{1}{2}$ | 85
$\frac{1}{2}$ = $\frac{1}{7}$ | $42\frac{1}{2}$
6
4 $73\frac{1}{2}$ price of 1*q.* 11*lb.*

CASE 4.

When the *tare* is rated at so much per cent on the *gross*, multiply the *gross* by the *tare* per cent, and divide by 100 for the *tare;* then calculate the amount as before.

EXAMPLE.

What is the value of 15*Hhds.* of Muscovedo sugar, weight gross 68*C.* 3*q.* 21*lb.* tare 12 per cent, at 14*cts.* per *lb.*

C. *q.* *lb.*
68 3 21
68
68
6884
21

$7721 \times \frac{12}{100} = 926$ 52 *lb.* tare..
926 52 deduct.

6794 48
14

$951, 22 71 Ans.

PRACTICAL EXAMPLES.

SHEWING THE USE OF DECIMALS.

ENGLISH MONEY.

As 20*s.* make a £. and 10 divides 20, giving a quotient of 2, 18*s.* are = .9 of a £. for $\frac{18}{20}$ = .9; hence then the following table will naturally arise, viz.

19*s.*[*]	18*s.*	17*s.*	16*s.*	15*s.*	14*s.*	13*s.*	12*s.*	11*s.*
.95£.	.9£.	.85£.	.8£.	.75£.	.7£.	.65£.	.6£.	.55£.

---

[*] Any number of shillings divided by 2, and a point placed to the left of the quotient, in unit's place, will give the decimal of a £.=in value to those shillings; if one shilling, a cypher must be placed next the decimal point; then to take the aliquot parts of the decimal of a shilling, the decimal of any smaller part is easily obtained.

10s.	9s.	8s.	7s.	6s.	5s.	4s.	3s.	2s.
.5£.	.45£.	.4£.	.35£.	.3£.	.25£.	.2£.	.15£.	.1£.

1s.	6d.	3d.	1½d.	¾d.
.05£.	.025£.	.0125£.	.00625£.	.003125£.

¼d.=.0010416+£. ½d=.0020833£. decimals.

If we now desired to reduce any number of shillings, pence and farthings to the decimal of a £. it is easily done by the above table.

EXAMPLE.

Let it be required to reduce 19s. 11¾d. to the decimal of a £.

19s.=.95 =19s.
6d.=.025 = ½
3 =.0125 = ¼
1½ =,00625 = ⅛
¾ =.003125 = 1/16
½=.0020833⅓= 1/24

.9989583⅓ Ans.

Otherwise,
19s.=.95
11¾d.
4

47+2=.049

.999 Ans. a little too much.

The pence and farthings reduced to farthings, increased by 1, when the farthings exceed 12, and by 2, when they exceed 37, comes near enough the truth for practice. They occupy the second and third places of the decimal.

6¼cts.=.0625 of a dollar.
10 =.1 of a dollar.
12½ =.125 of a dollar.
20 =.2 of a dollar.
25 =.25 of a dollar.
50 =.5 of a dollar.

$1,23¾ $1.2375

If one had occasion to calculate interest upon a sum of English money, it can be more conveniently done, when it consists of pounds, shillings, pence and farthings, by first reducing the shillings, &c. to the decimal of a pound: but as interest has not been yet discussed, we shall not offer any thing more upon this head.

Multiply 5£. 17*s*. 6*d*. by 7£. 12*s*. 6*d*.

17*s*.=.85£.	5.875	12*s*.=.6
6*d*.=.025	7.625	6*d*.=.025
.875	44.796875=	.625
decimal part of multiplicand.	£44 15 11¾	decimal part of multiplier.

If the price of 900 tons of iron was demanded when the ton sold for 39£. 18*s*. 9*d*. how should we tell it? Thus: £39

.9 =decimal of 18*s*.
.025 =decimal of 6*d*.
.0125=decimal of 3.

39.9375
900

35943.75.00=35943£. 15*s*. 0*d*. Ans.

Required, the price of 1700 tons of pigs at 13£. 19*s*. 6*d*. per ton. £13

.95 =decimal for 19*s*.
.025=decimal for 6*d*.

13.975
1700

23757.500=£23757 10 Ans.

Required, the price of 1000 barrels of oil, at 5£. 19*s*. 3*d*. per barrel. £5

.95 =decimal for 19*s*.
.0125=decimal for 3*d*.

5.9625
1000

5962.5000=£5962 10 A.

ASSESSMENTS,

OR METHOD OF LAYING TAXES.

RULE.

1. Divide the cents in the amount of tax to be raised, by the amount of dollars in the whole valuation, and the quotient will be the proportion for each dollar.

2. Multiply each man's valuation by the proportion which the dollar pays, and the thing is done.

EXAMPLES.

1. Suppose that the valuation of property in a township amounted to $400000, and it was required to raise from this amount a tax of $4000; how much should each dollar of the valuation pay?

Valuation. Assessment.

$400000 : 1 :: 4000 : \frac{4000}{400000} = \frac{4}{400} = \frac{1}{100} = 1\textit{ct.}$

2. Suppose the valuation was $50000, and tax to be raised, $700; what should the dollar pay?

Valuation. Assessment.

$50000 : 1 :: 700 : \frac{700}{50000} = \frac{7}{5} = 1\textit{ct.}\ 4\textit{m.}$ Ans.

By multiplying every man's valuation, by the proportion which the dollar pays, the product will be the amount of his tax.

APPORTIONING OF ASSETS.

Where there are several creditors, each having demands against one; but the effects of the debtor not being equal to the sum of the respective debts; how to apportion the amount of said effects, or to set out a proper share to each creditor.

RULE.

1. Divide the whole amount of the effects, by the whole amount of debts, and the quotient will give the part that each dollar should pay.

2. Multiply each creditor's part of the whole debt, by the part which is found for the dollar, and the product will be the proper share for such creditor.

EXAMPLES.

1. Suppose A owes to B $500.
to C 700.
to D 900.
to E 1000.
and to F 1900.

In all 5000.

And A is worth no more than $3000; what portion of this sum should each creditor have?

Operation. 5000 : 1 :: 3000 : $\frac{3000}{5000}=\frac{3}{5}=$60cts. the part which the dollar pays. Then,

$500\times.60=300.=$B's part.
$700\times.60=420.=$C's.
$900\times.60=540.=$D's.
$1000\times.60=600.=$E's.
$1900\times.60=1140.=$F's.

3000. Proof.

2. Suppose two men, A and B, purchase a plantation between them from C, containing 200 acres of land, for $30 per acre; A agrees to take a divide of the land (being the most valuable part) at $34 per acre, and B the remainder, at $26 per acre; when they ascertained their respective quantities of land, it was found that A's part contained 90 acres, and of course, B's 110 acres. How much must each man pay, so that the whole purchase money may be made up?

Purchase money=$6000.
A's part= 90*A*. at 34=3060.
B's =110 at 26=2860.

Amount of both=$5920.

Now we see that $5920 wants $80 of being enough; how much of this sum should each of them pay in order to make up the whole?

$$34+26=60 : \left\{\begin{matrix}34\\26\end{matrix}\right\} :: 80 : \left\{\begin{matrix}45\tfrac{1}{3}\text{ A's part.}\\34\tfrac{2}{3}\text{ B's.}\end{matrix}\right.$$

Proof 80

SIMPLE INTEREST.

Simple Interest is the *premium* or *hire* which the borrower pays the lender, for the use or loan of money.

The sum lent, is called *principal.*

The premium or hire, *interest*, or rate per cent.

The sum of principal and interest, *amount.*

Interest for 1 for 1 year, *ratio.*

Interest upon interest (generally calculated upon principal and interest both) is called *compound interest.*

RULE.

Multiply the principal by the rate per cent and time, divide by 100, and the thing is done. Or multiply by the ratio,* which is the most simple.

---

* When $100 or £100 are hired one year for $6 or £6 (or any other sum) $100 : $1 :: 6 : .06*cts.* the interest of $1. for 1 year=ratio; hence it is plain, that if any number of dollars be multiplied by .06, the product will be the interest of that sum for a year; or if any sum be multiplied by 6, and divided by 100, it gives the same; for the interest is the $\frac{6}{100}$ or $\frac{3}{50}$ parts of the principal, when the rate is 6 per cent, and time, 1 year.

TABLE OF RATIOS.

1	.01	4¼	.0425	7½	.075
1¼	.0125	4½	.045	7¾	.0775
1½	.015	4¾	.0475	8	.08
1¾	.0175	5	.05	8¼	.0825
2	.02	525	.0525	8½	.085
2¼	.0225	5.5	:055	8¾	.0875
2½	025	5¾	.0575	9	.09
2¾	.0275	6	.06	9¼	.0925
3	.03	6¼	.0625	9½	.095
3¼	.0325	6½	.065	9¾	.0975
3½	.035	6¾	.0675	10	.1
3¾	.0375	7	.07	10¼	.125
4	.04	7¼	.0725	10½	.15

EXAMPLES.

1. Find the interest of $375, 50 for one year, at 6 per cent.

Operation by rule 1st. $\frac{375,\ 50\times6}{100}=22,\ 53$ Ans.

Operation by rule 2d. $375,\ 50\times.06=22,\ 5300$ Ans.

2. Find the interest of 375£. 13*s*. 4*d*. for a year at 6 per cent.

By rule 1st.

```
     375 13 4
            6
    ─────────
100)2254 00 0
    ─────────
    22.54=
    £22 10 9⅗
```

By rule 2d.

```
£375 13 4
=375.666+
      .06
 ────────
 22.53996
or £22.54 very nearly.
```

The difference arising from the defect in the decimal .666+ being something less than 13*s*. 4*d*.

CASE 2.

To find the interest of any sum of money for years and months.

RULE.

Find the interest first for one year, multiply this by the number of years, take parts of the first year's interest for the given months, add the sum of those parts to the interest for years, and the thing is done.

EXAMPLES.

1. Find the interest of $1740, 40 for three years and nine months, at 6 per cent.

```
               $1740.40
                    .06
    M.         --------
    6=½)104.4240
                      3
               --------
               313.272
    3=½)  52.212
               26.106
               --------
Ans. $391.59
```

2. Find the interest of £3940 17*s*. 6*d*. for three years, nine months and a half, at 6 per cent.

	£3940	17	6		Or £3940.875
			6		.06=ratio.
M.				*M.*	
6=½)	236.45	5	0	6=½)	236.45250
			3		3
	709.35	15	0		709.3575
3=¼)	118.22	12	6	3=¼)	118.22625
½=⅙)	59.11	6	3	½=⅙)	59.113125
	9.85	4	4½		9.8521875
	£896.54	18	1½		£896.5490625
	20				20
	s.10.98				*s*.10.9812500
	12				12
	d.11.77				*d*.11.77500
	4				4
	qrs.3.10				*qrs*.3.100

Ans. £896 10*s*. 11*d*. 3.1*qrs*.

CASE 3.

To find the interest of any sum of money for years, months and days.

RULE.

Find the interest for the years and months, then say, as 365 : the given days : : the interest for one year to the interest required.

EXAMPLE.

1. Find the interest of $1000000 for 1 year, 9 months, and 19 days, at 6 per cent.

$$\begin{array}{rr} & 1000000 \\ & .06 \\ \hline \textit{M.} \quad 6 = \frac{1}{2} & 60000.00 \\ 3 = \frac{1}{2} & 30000 \\ & 15000 \end{array}$$

D. *D.*
365 : 19 : : 60000 : 3123.28¾ +

$108123.28¾ Answer.

If the above example had been calculated by the method usually practised in business (as shall be shewn below) a very material error would have resulted.

$$\begin{array}{lr} & 1000000 \\ & .06 \\ \hline 6\textit{M.} = \frac{1}{2} & 60000 \\ 3 \quad = \frac{1}{2} & 30000 \\ 15\textit{D.} = \frac{1}{6} & 15000 \\ 3 \quad = \frac{1}{5} & 2500 \\ 1 \quad = \frac{1}{3} & 500 \\ & 166\frac{2}{3} \\ \hline & \$108166.66\frac{2}{3} \end{array}$$

The interest found by the second method is too large; the difference being $43.38. This method of calculation is therefore very unsafe for large sums, being founded on the supposition that the year consists of 360 days.

TABLE OF DECIMAL PARTS OF A YEAR,

EQUAL TO ANY NUMBER OF DAYS.

Days	Decimal parts	Days	Decimal parts	Days.	Decimal parts
1	.00274	10	.027397	100	.273973
2	.005479	20	.054794	200	.547945
3	.008219	30	.082192	300	.821918
4	.010959	40	.109589	365	1.000000
5	.013699	50	.136986		
6	.016438	60	.164383	¼ year	= .25
7	.019178	70	.191781	½	= .5
8	.021918	80	.219178	¾	= .75
9	.024657	90	.246575		

2. Find the interest of $754070 for 2 years 11 months and 23 days, at 6 per cent, by the help of the preceding table.

754070 × .06 = 45244.20 = Interest for 1 year.
45244.20 × 2 = 90488.40 = 2 *Y.*
45244.20 ÷ 2 = 22622.10 = 6 *M.*
45244.20 ÷ 3 = 15081.40 = 4 *M.*
15081.40 ÷ 4 = 3770.35 = 1 *M.*

$$\overline{.054794+.008219}\times 45244.20 = 2850.98\tfrac{1}{4} = 23d.$$

$134813.23¼ Ans.

The same calculated otherwise.

Thus:

$$\begin{array}{r}754070\\ .06\\ \hline \left.\begin{array}{l}6M.=\frac{1}{2}\\ 4\quad=\frac{1}{3}\end{array}\right\} 45244.20\\ 2\\ \hline\end{array}$$

90488.40 for 2 years.
22622.10 for 6 mo.
1 *M.* = ¼)15081.40 for 4 mo.
D. *D.* 3770.35 for 1 mo.
365 : 23 :: 45244.20 : 2851.00 for 23 days

$134813.25 Answer.

The interest of any sum of money may be had, when the rate per cent is six, by multiplying the given principal by half the given months in time, and dividing by 100; or by multiplying by the ratio of half the given months to 100.*

Or, $754070
17.5=half the number of months in 2 years and 11 months.

3770350
1281919 } =interest for 2 years, 11 mo.
2851 =45244.20 × $\frac{23}{365}$ int. for 23*D.*

$134813.25 Answer.

What is the interest of £375 17*s.* 6*d.* for 3 years, 7$\frac{1}{2}$ months, at 6 per cent per annum?

375 17 6
6
2255 5 0
3
6765 15 0
1409 10 7$\frac{1}{2}$
£81,75 5 7$\frac{1}{2}$
20
s. 15,05
12
d. ,67
4
qrs. 2,70

Multiplier for 6*M.*=3
do. for 1*M.*=$\frac{1}{2}$
do. for $\frac{1}{2}$*M.*=$\frac{1}{4}$

Or the interest of $\frac{1}{2}$*M.* may be had by taking $\frac{1}{2}$ of the interest found for a month.

1*M.*=$\frac{1}{2}$)375 17 6
3
1127 12 6
$\frac{1}{2}$*M.*=$\frac{1}{2}$) 187 18 9
93 19 4$\frac{1}{2}$
1409 10 7$\frac{1}{2}$

Ans. £81 15*s.* 0*d.* 2.7*qrs.*

---

* Let us suppose $100=principal, $6.=its interest for 1 year=12 months, P any other principal, and T any other time, 100 : 6 :: P : $\frac{6P}{100}$ and 12*M.* : T :: $\frac{6PT}{100}$

2. Find the interest of 3400 dollars for 19 months, at 6 per cent.

Operation. $\frac{3400 \times 9.5}{100} = 323.$ Answer.

*But since $100 : 9.5 :: 1 : .095$; the same result is had by multiplying by .095;

$3400 \times .095 = \$323.000$, the same as before.

Interest calculated by the Double Rule of Three.

Find the interest of 940 dollars for 29 days, at 6 per cent per annum.

	D.	Days.	In.
Say if	100 ..	365 ..	6
	940 ..	29 ..	

Operation. $100 \times 365)940 \times 29 \times 6(4.48.3+$ Ans.

It is a custom with merchants to calculate interest on all balances which remain due upon credit sales, after a certain time. Generally, credit extends to two, four, and six months. After these periods of credit have gone round, interest is calculated for the time which elapses between that and the time of settlement, also upon each intermediate payment, from the time when made to that of settlement.

A purchased goods to the amount of 3740 dollars, at 2 months' credit, 5740 dollars at 4, and 7500 dollars at 6 months; date of purchase, April 1, 1820. On June first, paid 1975 dollars, September 1, 4000,

---

* This is the same thing as multiplying the principal by half the time in months, and then dividing the result by 100.

dollars, January 1, 1821, paid 3000 dollars. On April 1, 1821, settled and balanced accounts; how did it stand?

Due June 1, 1821,	$3740
Paid same date,	1975
	1765
Int. for 3*M*. due,	26.47½
2d. payment due,	5740
Int. for 1*M*. due,	28.70
	7560.17½
Paid Sept. 1.	4000
Jan. 1, 1821.	3560.17½
Interest for 4*M*.	71.20
Oct. 1, due,	7500
Jan. 1, 1821, Int. due for 3*M*.	112.50
	11243.87½
Paid Jan. 1, 1821,	3000
	8243.87½
April 1, 1821, Int. for 3*M*. due,	123.65¾
Due at settlement	$8367.53¼
*1st. method,	8367.53¼
2d. method,	8362.85
Defference,	$4.68¼

But if interest had been reckoned on the payments, how would the account then have stood?

Interest on	3740
for 10 months,	187
Interest on	5740
for 8 months,	229.60
Interest on	7500
for 6 months,	225
A'mt of debt & int.	$17621.60
1st. payment, June 1, 1820.	1975
Interest 10 months,	98.75
2d. payment Sept. 1,	4000
Interest 7 months,	140
3d. payment Jan. 1,	3000
Interest 3 months,	45
Amount of payments & interest	$9258.75*

17621.60 debt.
9258.75 credit.

$ 8362.85 balance by second method.

---

* The first method gives an advantage to the creditor; because the payment is always applied first to discharge the interest, and then to the discharge of the debt, as in justice it ought; because the money was not paid when due. The interest is therefore due at any time.

Having said what I consider sufficient upon the calculation of common or simple interest, I have subjoined something upon bank interest.

The mode in banks, is to calculate their interest upon the month of 30 days. Finding the interest of the sum, from the date of the note to the time it becomes due, including the days of grace (which are generally three,) the interest thus found, is called the discount. A note dated on March 5th, drawn for 60 days after date, would actually expire, or become due, on the 4th of May; but the bank granting three days of grace, causes it to fall due on the 7th of May; which is the day that will be marked on the notice, that is always given to the person for whom the note has been discounted. A similar procedure takes place: let the note be drawn for any number of days whatever, on the renewal of the note the first and last days are both included; which makes, in addition to the three days of grace, another day; (that on which the money is paid to the drawer, being recognised as a new transaction.) If a note is drawn for 30, 40, 60, &c. days, interest must be calculated for 34, 44, 64, &c. days; which plainly shews, that banks have more profit on notes renewable every 30, 40, &c. days, than 60, 80, &c.

As 100 dollars gives 6 dollars interest in 360 days, it will give a proportional interest for any other number of days, of which 360 is a multiple.

		Reckoning for 3 days' grace.	Reckoning for 4 days; 3 of grace, 1 paying money.
$100 for 360 days, gives	6.00	6.05	6.07
100 for 180 days, gives	3.00	3.05	3.07
100 for 90 days, gives	1.50	1.55	1.57
100 for 60 days, gives	1.00	1.05	1.07
100 for 30 days, gives	.50	.55	.57
100 for 15 days, gives	.25	.30	.32

Bank interest is calculated by the following

RULE.

Multiply by the given number of days for which the note was drawn, (including days of grace and the day on which the money was paid to the drawer) and divide by 360.

EXAMPLE.

What is the interest, or bank discount, of a note of $100, drawn for 60 days?

Days. *Days.*
360 : { 60
3 days grace.
1 day, when the money was paid.
—
64 :: 6, interest of $100 for 360*D.*
6 : to the int. or dis. required.
360)384(1.06⅔ Ans.
360
——
2400
2160
——
$\frac{240}{360}=\frac{2}{3}$

But in all cases where the interest comes out a fraction, it is made even; the interest would in that case be $1.07.

But the bank method of calculation is shorter, and may be easily known, from doing the same example over again.

For 60 days - - - - $1.00
3 days grace.
1 day of payment.
—
4 being equal to the 15th part of 60, such part of $1. is that required for 4 days, .07
——
Ans. $1.07

If for 95 days,

Say	60 days	$1.00
	30	.50
	6	.10
	3	.05

Int. for 99 days = 1.65

If for 120 days.

Say for 60 days	1.
60	1.
4	.07
Int. for 124 days	$2.07

If for 116 days.

Say for 60 days	1.
60	1.
Interest for 120	$2.

Another rule is as follows: from the sum, or amount of the note, cut off two figures to the right hand, divide by 15, add the quotient, and the thing is done.

EXAMPLES.

15)1.00
.06⅔

1.06⅔ in bank 1.07

If for 90 days.
Add .50 for 30 days.
And 1.07 for 64

$1.57 for 94 days.

Suppose for 30 days.
Put down 1. for 60 days.
.07 4 days.

1.07 for 64 days.
Deduct .50 for 30 days.

.57 for 34 days.

In the state of Pennsylvania, Chief Justice M'Kean, in 1785, fixed the following rule for calculating interest, (laid down in 1st Dallas, page 124,) viz. "That the interest of the money paid in before the time, be deducted from the interest of the whole sum due at the time appointed by the instrument for making the payment."

EXAMPLE.

John Stiles, in account with John Nokes,

Dr.		*Per contra, Cr.*	
1820,		1819;	
Jan. 1. To his bond	$100.	July 1. By cash	$50.
To one year's interest,	6.	1820,	
		Jan. 1. By interest of $50 paid as above,	1.50*
		By balance,	54.50
	$106.		$106.

If the $50 dollars which was paid July 1, had been deducted from $103, the whole principal and interest (supposing it to be then due,) there would have remained $53, which, at the end of the year with its interest, would make $54.59, which is 9 cents more than just; because no part of the interest was payable until the end of the year. If the bond had been for $1000000, the gain to the obligee $900 over and above the legal hire.

If money becomes due, and interest accrues thereon, it is an established rule, that if part of the money be paid after such circumstance occurs, to discharge the interest then due first, and apply the residue towards the discharge of the principal; the remainder becomes principal, on which interest may be calculated to the time of next payment: but if the payment be too small to extinguish the interest due at the time, interest is only to be reckoned on the residue of the principal, and not on the part of interest which remains unsatisfied; otherwise, it would be compound interest.

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* It is strictly correct to give Stiles credit for the interest of the $50 paid in, as he might have had it from another; and it is no loss to Nokes, for it brings him as much as Stiles' credit.

TO FIND THE INTEREST OF ENGLISH MONEY IN FEDERAL.

£6. being equal to $16., and $16.=the interest of £100. for one year, at 6 per cent, the interest of any number of £. may be easily brought out in dollars and cents, by the following

RULE.

Multiply the given £. by 16, and cut off two places to the right hand, for cents, and the thing is done.

EXAMPLE.

Required the interest of £79. for 1 year, at 6 per cent, in dollars and cents.

£.	£.	D.	D.
100 :	79 : :	16 :	12.64 Ans.

Rules for 3, 4, 5, &c. or 3½, 3¾, &c. per cent, can be easily deduced from that given for 6 per cent, by the following proportion:

As £100,
Is to any principal in pounds;
So is the rate per cent, in Federal money,
To the interest of the same principal, for one year, in Federal money.

A TABLE

SHEWING THE NUMBER OF DAYS, FROM ANY DAY IN ANY MONTH, TO THE SAME DAY IN ANY MONTH FOLLOWING.

	From January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
January.	365	334	306	275	245	214	184	153	122	92	61	31
February.	31	365	337	306	276	245	215	184	153	123	92	62
March.	59	28	365	334	304	273	243	212	181	151	120	90
April.	90	59	31	365	335	304	274	243	212	182	151	121
May.	120	89	61	30	365	334	304	273	242	212	181	151
June.	151	120	92	61	31	365	335	304	273	243	212	182
July.	181	150	122	91	61	30	365	334	303	273	242	212
August.	212	181	153	122	92	61	31	365	354	304	273	243
September.	243	212	184	153	123	92	62	31	365	335	304	274
October.	273	242	214	183	153	122	92	61	30	365	334	304
November.	304	273	245	214	184	153	123	92	61	31	365	335
December.	334	203	275	244	214	183	153	122	91	61	30	365

USE OF THE PRECEDING TABLE.

Required, the number of days from the 9th of May to the 29th of December following.

Under May, and opposite December,	214
From the 9th to the 29th,	20
Answer,	234

PROOF.

May, - - -	22
June, - -	30
July, - -	31
August, - -	31
September, -	30
October, -	31
November, -	30
December, -	29
	234

If the days be different, it is only adding or subtracting their inequality to or from the tabular number. In leap years, if the end of February be in the time, one day must be added on that account.

In Simple interest, $p.$ represents principal, $t.$ time, $r.$ ratio, and $a.$ amount. From these the four following theorems arise, which shew every possible case of Simple Interest:

$$\text{1st. } ptr+p=a. \quad \text{2d. } \frac{a}{tr+1}=p. \quad \text{3d. } \frac{a-p}{tp}=r.$$

$$\text{4th. } \frac{a-p}{rp}=t.$$

EXAMPLES.

1. Required, the amount of $375.50, at 6 per cent per annum, for 6 years.

Operation. $375.50\times.06\times6+375.50=510.68$ Ans.

2. What principal, at 6 per cent per annum, will, in 6 years, amount to $510.68?

Operation. $\frac{510.68}{1+.06\times6}=375.50$ Ans.

3. At what rate per cent will £375.50 amount to $510.68 in 6 years time?

Operation. $\frac{510.68-375.50}{375.50\times6}=.06$ Ans.

4. In what time will $375.50 amount to $510.68, at 6 per cent per annum?

Operation. $\frac{510.68-375.50}{375.50\times.06}=6$ years.

TABLE OF DECIMAL PARTS

FOR EVERY DAY IN THE 12*th* PART OF A YEAR WHICH CONSISTS OF 365.25, OR 365¼ DAYS.

Days.	Decimal parts.	Days.	Decimal parts.	Days.	Decimal parts.	Days.	Decimal parts.	Days.	Deicmal parts.
1	.033	7	.23	13	.427	19	.624	15	.821
2	.066	8	.263	14	.46	20	.657	26	.854
3	.098	9	.296	15	.493	21	.69	27	.887
4	.131	10	.328	16	.526	22	.723	28	.92
5	.164	11	.361	17	.558	23	.756	29	.953
6	.197	12	.394	18	.591	24	.788	30	.986

By help of the above table, the interest of any sum may be easily had for any number of months, or months and days, at 6 per cent per annum. By multiplying the number of months (or months and parts of a month, answering to the given number of days in the table,) cutting off one figure more in the product than the rule of decimals require, the result will be the interest in shillings and decimal parts of a shilling, if the money be English; but if Federal, cut off the proper number required by decimals, and divide by 200, and the result will be the interest in dollars and cents.*

EXAMPLES.

1. Find the interest of $1347.50 for 4 months and 23 days, at 6 per cent per annum.

Tabular number answering to 23 days=.756 of a month, and 4.756=time.

Operation. $\frac{1347.50\times4.756}{200}=32.04.3+$ Ans.

2. Find the interest of $1000000 for 17 days, at 6 per cent per annum.

Operation. $\frac{1000000\times.558}{200}=2790.$ Ans.

---

* This rule may be demonstrated as follows, viz. First, find in what time $100 principal will give $100 interest, thus; $\overset{p.}{100}\times\overset{in.}{6} : \overset{p.}{100}\times\overset{in.}{100} :: \overset{Y.}{1} : 16\frac{2}{3}$ years.

Or, $\overset{p.}{100}\times\overset{in.}{6} : \overset{p.}{100}\times\overset{in.}{100} :: \overset{M.}{12} : \overset{M.}{200}$. And by reversion:

$\overset{M.}{200} : \overset{M.}{12} :: \overset{p.}{100}\times\overset{in.}{100} : \overset{p.}{100}\times\overset{in.}{6}$. Wherefore,

$\overset{M.}{200} : \overset{M.}{12} :: \overset{p.}{100} : \overset{in.}{6}$, and $\overset{M.}{200} : \overset{M.}{1} :: \overset{p.}{100} : \overset{cts.}{50}$ the interest of $100 for 1 month, at 6 per cent. Therefore, the reason for dividing by 200 in the above is evident; any number of months and decimal parts of months, being the same way proportional to the interest for the time

GENERAL RULES AND THEOREMS

FOR CALCULATING SIMPLE INTEREST.*

We know that any sum of money being upon simple interest at 6 per cent. will become double in 16 years and 8 months, or in 200 months. From this principle, we obtain a lively method for calculating interest at 6 per cent. and deduce the following concise and elegant

THEOREM:

Putting P = any given principal, and T any given time in months, then we have

$$200 : T :: P : \frac{T}{2} \times \frac{P}{100} = \frac{TP}{200}$$

This theorem expressed in words, reads thus:—

Multiply the given principal by half the given time in months, cutting off two figures on the right hand for decimals, and you have the interest required.

Or, it may be more convenient to express it thus:

Multiply the given principal by the given time in months, and divide by 200.

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* This article, containing three theorems for calculating simple interest, was drawn up and handed to me, by my good friend John Capp, Esq. of Harrisburg. The first and third of these theorems, have not, I believe, been published in this form, in any book on arithmetic. Some parts of the second theorem, may be found in several works of arithmetic; but I have not seen all its parts brought together in any book before, with their several demonstrations. It may be further remarked, that these three theorems embrace, with few exceptions, the most general cases of simple interest; and that they furnish, generally speaking, the shortest rules that can be formed.

THEOREM 2.

Because 1 month is $\frac{1}{12}$ of a year, $1\frac{1}{2}m.=\frac{1}{8}$, $2m.=\frac{1}{6}$, $3m.=\frac{1}{4}$, $4m.=\frac{1}{3}$, $6m.=\frac{1}{2}$, $8m.=\frac{2}{3}$, $9m.=\frac{3}{4}$, and $10m.=\frac{5}{6}$ of a year, the following general rule for calculating simple interest, on any given principal, for any given number of months, at the following rates of interests, becomes evident:

$$\begin{array}{l}\text{When the}\\ \text{rate per cent.}\\ \text{per annum is}\end{array}\left\{\begin{array}{c}1\\1\frac{1}{2}\\2\\3\\4\\6\\8\\9\\10\end{array}\right\}\begin{array}{l}\text{Multi-}\\ \text{ply the}\\ \text{given}\\ \text{princi-}\\ \text{pal by}\end{array}\left\{\begin{array}{c}\frac{1}{12}\\ \frac{1}{8}\\ \frac{1}{6}\\ \frac{1}{4}\\ \frac{1}{3}\\ \frac{1}{2}\\ \frac{2}{3}\\ \frac{3}{4}\\ \frac{5}{6}\end{array}\right\}\begin{array}{l}\text{Part of the given}\\ \text{number of } \textit{mon.},\\ \text{cutting off two fi-}\\ \text{gures on the right}\\ \text{hand for decimals,}\\ \text{and the result will}\\ \text{give the interest}\\ \text{of that principal}\\ \text{for the given time.}\end{array}$$

DEMONSTRATION. Denoting any given principal by P, and the given time by T, and supposing the rate per cent to be 6; then it will be,

As $100 : P :: 6 : \frac{6P}{100}$ the interest of P for one year. And again,

$$\text{As } 12 : T :: \frac{6P}{100} : \frac{6T}{12}\times\frac{P}{100}=\frac{T}{2}\times\frac{P}{100}.$$

This theorem expressed in words, reads thus:

Multiply the given principal by half the given time in months; cutting off two figures on the right hand, and you will have the interest required.

In a similar manner, the rules at the several other rates may be demonstrated.

Again; if the rate per cent be 5, we have

$100 : P :: 5 : \frac{5P}{100}=\frac{P}{20}=$ the int. of P for 1 year.

Hence, by dividing any principal by 20, we obtain the interest of that principal for one year.

When interest is at 7, 10, 11, &c. per cent. it will be most convenient, first to find the interest at some rate which is an aliquot part of 12, and then take parts for the remaining part of such rate.

The interest may also be had when the rate is 7, 11, &c. per cent. by multiplying the principal by the rate per cent. and by the time in months; and dividing the result by 12, and by 100. Because,

$100 : P :: R : \frac{PR}{100}$ = the interest of P for one year; R denoting the rate per cent. And again,

$12 : T :: \frac{PR}{100} : \frac{PRT}{12 \times 100}$ = the interest of P, for the time T, at the rate R.

Observing always, that if any of the quantities represented by P, R, or T, be divided by 12, or by 100, before they are multiplied together, the work will be shortened.

THEOREM 3.

The Legislature of Pennsylvania, in passing laws to incorporate and charter the several companies for the purpose of *banking*, in this state, having authorised these several institutions to exact from their customers, who borrow money from them, on all loans at the rate of *one per cent.* for 60 days; and interest is now universally charged at this rate in all the banks of the state.

Hence, this rule of calculating interest has become the law of the state; and certainly, no reason exists, why *every citizen* of the state should not have the same *privilege* that is specially granted to these *banking institutions;* especially when the time is less than a year.

Hence, we deduce this useful theorem, for calculating interest for days, at 6 per cent.

Putting P=any given principal, and T=any given time in days; then we shall have,

$60 : T :: 1 : \frac{T}{60}$=the interest of $100 for the given time. And again,

$100 : P :: \frac{T}{60} : \frac{T \times P}{6000} = \frac{T}{6} \times \frac{P}{1000}$=the interest of P for the whole time.

Now this theorem expressed in words, reads thus:

Multiply the given principal by the given days, divide the product by 6, and cut off three places for decimals on the right hand, and you have the interest required.

NOTE. It is evident by inspection, that, when *T*, the number of the given days, or P, the principal, happen to be either a part or a multiple of the denominator 6, or 100, the work will be shortened, by dividing such fators as will measure each other, by any common divisor.

It is also evident, that this theorem furnishes the shortest general rule possible, for calculating simple interest for days.

COMPOUND INTEREST.

Compound Interest, or interest upon interest, is scarcely used in business; we shall therefore give but one or two examples, to shew how it is calculated.

EXAMPLE.

What is the compound interest of $3475 for 3 years, at 6 per cent per annum?

3475		
.06	ratio for simple interest.	
208.50	Interest for 1st year,	208.50
3475.		
3683.50		
.06		
221.010	Interest for 2d year,	221.01
3683.50		
3904.51		
.06		
234.2706	Interest for 3d year,	234.2706
3904.51		663.7806
4228.7806 3475.	Simple interest for the same time only	625.50
$ 663,7806 Ans.	Difference,	$38.2806

By the compound ratio,

$3475 \times 1.06 \times 1.06 \times 1.06 = 4138.7806 =$ amount,

and $4138.7806 - 3475 = 664.7806 =$ interest.

If $r = 1.06$ amount of 1£. or 1$. for 1 year,

$p =$ principal,

$i =$ interest,

$t =$ time,

$a =$ amount,

The following theorems shew the method of resolving all the cases of Compound interest, except t; which must be found by logarithms, viz.

1. $pr^t = a$. 2. $pr^t - p = i$. 3. $\frac{a}{r^t} = p$. 4. $\left.\overline{\frac{a}{p}}\right|^{t} = r$.

To find the interest of \$3475 according to theorem 2d, on the preceding page.

$$3475\times\overline{1.06}\,|^{3}-3475=3475\times1.191016-3475=$$ \$663.7806 Answer.

To find the amount, $\$3475\times\overline{1.06}\,|^{2}=4138.7806$ Ans.

To find the principal, $\frac{4138.7806}{\overline{1.06}\,|^{3}}=\frac{4138.7806}{1.191016}=3475$ Ans.

To find the ratio, or rate per cent, $\sqrt[3]{\frac{4138.7806}{3475}}=1.06=$ amount of \$1 for 1 year, and $1.06-1=.06=\frac{6}{100}=$rate per cent=6 for 100.

To find the time, $\frac{\text{Log. }4138.7806-\text{Log. }3475}{\text{Log. }1.06}=3.$ Ans.

For $\frac{\text{Log. }a.-\text{Log. }p.}{\text{Log. }R.}=t.$ as is shewn further on.

* * *

REBATE OR DISCOUNT.

Discount is an allowance made for the payment of money before it is due; which allowance is less than common interest for the time, by the interest of the interest.

The balance, after discount is deducted, is called the present worth.

If the present worth be put to interest, at the rate per cent and for the time, the amount will be equal to the sum which would have then been due by the obligation.

To find the discount of any sum of money for one year,

RULE.

Multiply the sum by the rate per cent, and divide by a hundred more the rate per cent, and the thing is done.

EXAMPLES.

1. Find the discount of $375, due one year hence, at 6 per cent.

106 : 6 : : 375 : 21.22$\frac{34}{53}$ Ans.

2. Find the discount of $3754.40, due 1 year hence, at 6 per cent. Ans. 212.51\frac{17}{53}$.

To find the present worth of any sum for one year,

RULE.

Multiply the sum by 100, and divide by 100 more than the rate per cent, and the thing is done.

EXAMPLES.

1. Find the present worth of $1760, due 1 year hence, at 6 per cent.

106 : 100 : : 1760 : 1660.37$\frac{39}{53}$ Ans.

2. Find the present worth of a bond for $1000, due 1 year hence, at 6 per cent. Ans. 943.39\frac{33}{53}$.

To find the present worth of a note or bond, due 2, 3 or more years hence,

RULE.

Divide the given sum by the product of the time and ratio more 1, and the thing is done.

EXAMPLES.

1. Find the present worth of $4757.60, due 2 years hence, at 6 per cent.

.06 ratio.
2 time.
———
.12
Add 1.
———
1.12)4757.60(=4247.85$\frac{5}{7}$ Ans.

2. Find the present worth of $3847, due 5 years hence, at 6 per cent. Ans. 2959.28\frac{1}{13}$.

To find the present worth of a bond, due at the end of any number of years and months, at any rate per cent.

RULE.

Multiply the number of months in the given time by the rate per cent, divide by 12; add 100 to the quotient, by this divide the given sum multiplied by 100, and the thing is done.

EXAMPLES.

1. Find the present worth of $450, due 1 year and 9 months hence, at 6 per cent.

Operation. 21 months in given time.
6 rate.
———
12)126
———
10.5 = quotient.
100
——— *D. cts. m.*
10.5 + 100 = 110.5)450 × 100(407.23.9 + Ans.

2. Find the present worth of \$995, due 1 year and 11 months hence, at 7 per cent.

Ans. 877.28+.

Solution. First, time$=1\frac{11}{12}$years$=\frac{23}{12}$ years, and the ratio$=.07$; hence then $\frac{23}{12}\times .07=\frac{1.61}{12}$, to which if a unit be added, we have $1+\frac{1.61}{12}=\frac{13.61}{12}$ a divisor, and $\frac{995}{\frac{13.61}{12}}=\frac{995\times 12}{13.61}=\frac{11940}{13.61}=877.28+$ Ans.

This method of solution shall be demonstrated farther on.

If we divide 100 by any rate per cent, the quotient will shew a number equal to the number of years in which any sum will double itself at such rate.

EXAMPLES.

At 1 per cent	$\frac{100}{1}=$	100 years	$=$ 1200 months.
2	$\frac{100}{2}=$	50	$=$ 600
3	$\frac{100}{3}=$	33 4*M.*	$=$ 400
4	$\frac{100}{4}=$	25	$=$ 300
5	$\frac{100}{5}=$	20	$=$ 240
6	$\frac{100}{6}=$	16 8*M.*	$=$ 200
7	$\frac{100}{7}=$	$14\frac{2}{7}$	$=$ $171\frac{3}{7}$
8	$\frac{100}{8}=$	12.6	$=$ 150
9	$\frac{100}{9}=$	$11\frac{1}{9}$	$=$ $133\frac{1}{3}$
10	$\frac{100}{10}=$	10	$=$ 120
&c.	&c.	&c.	&c.

If it was required to find the discount of any sum, at any rate per cent, for any number of years and months,

RULE.

Add to the number of months corresponding with the rate per cent in the foregoing scheme, the given time reduced to months, for a divisor; multiply the given sum by the time in months, and divide by said divisor, and the quotient will be the discount required.

EXAMPLES.

1. Find the discount of $1150, due 20 months hence, at 6 per cent.

Months opposite 6 = 200
Given months = 20

Divisor, 220)1150 × 20)$104\frac{6}{11}$. Ans.

2. Find the present worth of $750.60, due 7 years and 11 months hence, at 8 per cent.

Ans. $291.04+.

3. Find the present worth of $1150, due 20 months hence, at 6 per cent.

The time is 20 months $=1\frac{2}{3}$ years $=\frac{5}{3}$ years; *now*, $\frac{5}{3}\times .06=\frac{30}{3}=\frac{.3}{3}=.1$, and $1+.1=1.1$ is a divisor for 1160, and $\frac{1150}{1.1}=\$1045\frac{5}{11}$, present worth required. The time in months can be easily reduced to the vulgar fraction of a year, or to a fraction which will express the time; this multiplied by the rate per cent and 1 added, or a unit added to the product, will be a divisor in all cases for the sum to be discounted, which, divided by it, will be the present worth.

BROKAGE.

1. Brokage is an allowance made to persons called brokers.

Brokers, are all such persons as buy and sell bank notes, specie, &c.; and such as find customers and sell the goods of other men, at a certain rate per cent, for their commission.

2. The value of bank notes, specie, &c. is found by the rules of Rebate or Discount, already taught.

3. When a sum of money, of any kind, is sold or bought at any rate per cent, discount or advance, and paid for in other money; the same sums, at the rates agreed upon, will reciprocally purchase or pay for each other, if calculated by the rule of discount.

EXAMPLES.

1. Find how much par paper will pay for $1275, at 2½ per cent discount.

100
2.5

102.5 : 100 :: 1375 : 1341.46.3+ Ans.

2. Find how much paper, at 2½ per cent discount, will purchase $1341.46.3 at par.

$\overline{1341.46.3 \times .025} + 1341.46.3 = 1374.999+$ or,
$1375. Ans.

The above, calculated as factorage, or commission, (which is the usual method in practice.)

$1375 - \overline{1375 \times .025} = 1340.62\frac{1}{2}$. Ans.

$1340.62\frac{1}{2} + \overline{1340.62\frac{1}{2} \times .025} = 1373.78+$. Which results plainly shew, that if a person buy or sell notes at discount or advance, the advantage is by no means reciprocal, if calculated as next above.

The true principle of money brokerage is, always to give as much of one kind of money for another, as that other would purchase of it, at the fair discount agreed upon.

FACTORAGE OR COMMISSION.

Factors, or commission merchants, are such persons as buy and sell goods for others, at so much per cent, according to the custom of the country or place where they reside. Their allowances are sometimes calculated as simple interest for a year, and sometimes by the rule of discount.

EXAMPLES.

1. Sold goods to the amount of $104000, at 1½ per cent commission: what does my factorage amount to? Operation. 104000 × .015* = 1560. Ans.

2. Bought a quantity of merchandize, amounting to $1000000, at ½ per cent commission; what is my factorage? Ans. $5000.

3. Sold goods to the amount of $75400, at 2½ per cent commission; what will factorage amount to, if calculated by the rule of Discount? Ans. $1839.03+

4. What is the amount of brokage on $74948, at ¾ per cent? Ans. $562.11

By the following scheme, the difference between Discount and Interest may be easily seen.

	Interest.	Discount.	
$1000, 1 *M.* at 6 per cent,	$5.	$4.97,	difference .03
2 - -	10.	9.90,	.10
4 - -	20.	19.78,	.22
8 - -	40.	38.46,	1.54
12 - -	60.	56.61,	3.39
60, or 5 years,	300.	230.77,	69.23
120, or 10,	600.	375.	225.

From the above it is evident, that every prudent person will take care to pay but the discount (when necessary) in place of interest.

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* Ratio = rate per cent ÷ 100. If $6 be allowed for the use of $100, 1 year, the amount at the end of the year would be $106; if only $1, the amount would be 1.06, and the ratio = $\frac{6}{100}$ = .06.

EQUATION OF PAYMENTS.

Equation of Payments is a method of finding when any number of notes or bonds, due at different times, may be all paid at once, without loss to debtor or creditor.

RULE.*

Multiply each payment by its time; divide the sum of the products thence arising, by the sum of all the payments, and the quotient will be the equated time required.

EXAMPLES.

1. A owes B a bond for $100 dollars, due two months hence, and one for $500, due 19 months hence; what would be the equated time for paying them both at once?

$$\begin{array}{rcl} \$100\times 2M. & = & 200 \\ 500\times 19 & = & 9500 \end{array}$$

Aggregate or sum 6|00 of payments.)97|00 aggregate of products.

$16\frac{1}{6}M.$ Ans.

Or, $\frac{1\times2+5\times19}{1+5}=16\frac{1}{6}$ as before; for 1 and 5 are to each other, as 100 to 500.

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[*] Much contention has arisen about the correctness of this rule. The rule presumes, that the interest of all the sums which are paid after they become due, is equal to the interest of the sums which are paid before they become due; and is susceptible of plausible demonstration, which is unnecessary to exhibit here, as any one can easily convince himself by a simple process.

My own opinion is, that the equated time should be so adjusted, that the interest of all the sums paid after they

2. A man purchased a plantation for $10000, payable in annual payments of $1000 each; but they afterwards agreed to settle it all at once, by fixing upon a time that neither should be a loser. Required, the time? Ans. 5½ years.

3. A merchant bought a quantity of goods, amounting to $10000, to pay in equal proportions, at 2, 4, 6, and 9 months, but afterwards agree to pay the whole in 5 months; which of them, debtor or creditor, had the advantage?

By rule, 5¼ months is the equated time; but *if* payment be made at the end of five months, the creditor has the advantage; for his money will bring him more interest than the discount, (forbearing 5¼ months) will serve the debtor.

4. A owes B 5 bonds, for $945 each, payable at 3, 9, 11, 19, and 29 months; what time might they all be paid at once? Ans. 14⅕ months.

5. Suppose a debt was discharged in the following manner; ¼ in hand, ⅜ in 3 months, ¾ of $\frac{9}{10}$ in 5 months, and the residue in 11 months; might it have been sooner discharged without loss to debtor or creditor?

It is a rule in law that real property cannot be condemned and sold for debt, if it will rent for as much as will pay the debt and costs, with interest, in seven years. To find the annual rent which any property should bring, in order to discharge a given debt in seven years; or, to know whether a property (according to law) should be condemned for any debt, or not, is a matter but superficially understood by most men.

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become due, should be precisely equal to the discount of all the sums which are paid before they become due. This opinion can be maintained by equally plausible demonstration. Here I shall drop the subject without further comment.

In order to remove this difficulty, I will here insert a correct theorem, taken from page 23, Analyst.*

1. Say amount of debt, interest, and costs, now due against a property=1000, and time seven years; the lawful rate per cent being 6, amount of $1 for 1 year=1.06; our theorem will then literally stand, $\$1000 \times \frac{\overline{1.06-1} \times \overline{1.06}^{\,7}}{1.06^7-1} = 1000 \times .17913$ $=179.13$ Ans. Now, because .17913 comes so very near to .18, we may safely reject the former and use the latter.

If any debt be multiplied by 18, and two places cut off to the right hand of the product, it will give the answer near enough for practice. Take the first example, 1000 × 18=18000, two figures being cut off, leaves 180, which is only 87 cents too much, in a debt of $1000.

2. What annual rent would be sufficient to pay the amount of debt, interest and costs (upon a property) amounting in the whole to $4000, in 7 years?

$4000 \times .18 = 720$ Ans.

3. A property which rented annually for $359, was condemned by a court of inquiry, for an amount of debt against it of $2300; was the condemnation lawful? $2300 \times .18 = 414$. Now, because this sum exceeds 359 by $55, the condemnation was lawful.

4. A property, worth an annual rent of $750, has against it $3000 of debt, should it be condemned?

$3000 \times .18 = 540$. Now, because this is less than 750, by a considerable sum, the property should not be condemned.

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* General form of the theorem is as follows, viz. $x = a \times \frac{\overline{r-1} \times r^n}{r^n-1}$. In which x=sum which may be drawn daily, weekly, monthly, quarterly, or yearly; a=sum forborne at interest; r=amount of $1 for the time of drawing, one day, one week, one month, or one year; n=whole time.

BUYING AND SELLING STOCKS.

Stocks are funds established by government, or individuals, in a corporate capacity; the value of which is fluctuating.

The value of stock, at any rate per cent above or below par, is found by the following

RULE.

Multiply by the rate per cent, and divide by 100, or point off two decimal places to the right of the rate per cent, (it then becomes a ratio;) multiply the stock by such ratio, and the thing is done.

EXAMPLES.

1. What is the amount of 3756 dollars national bank stock, at 130 per cent.

$3756 \times 1.30 = 4882.80$ Ans.

Second method. $25 = \frac{1}{4} \,|\, 3756$

$5 = \frac{1}{5} \,|\, 939$

187.80

\$4882.80 Ans.

2. What is the value of 6000 dollars worth of bank stock, at 93 per cent? Ans. \$5580.

3. What amount of stock, at 87½ per cent, can be purchased for 9000 dollars? Ans. \$10285.71+

4. What is the amount of 8750 dollars of 8 per cent stock, at 115 per cent? Ans. \$10062.50.

5. What is the amount of 9000 dollars of 6 per cent stock, at 87½ per cent? Ans. \$7875.

6. What is the amount of 5700 dollars 3 per cent stock, at 93¾ per cent? Ans. \$5343.75.

NOTE. There is what is called
8 per cent stock,
6 per cent stock,
3 per cent stock,
deferred stock,
These stocks are instituted for the support of government.

TABLE OF POWERS.

Roots,	1	2	3	4	5	6	7	8	9
Squares,	1	4	9	16	25	36	49	64	81
Cubes,	1	8	27	64	125	216	343	512	729
Biquadrates,	1	16	81	256	625	1296	2401	4096	6561
Sursolids,	1	32	243	1024	3125	7776	16807	32768	59049
Square cubes,	1	64	729	4096	15625	46656	117649	262144	531441
Second sursolids,	1	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrates squared	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
Cubes cubed,	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
Sursolids squared,	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401

INVOLUTION,

OR THE METHOD OF RAISING POWERS.

1. The powers of any number, are the successive products arising from unity, continually multiplied by such number.

2. The first power of any number is, unity multiplied by such number.

3. Unity multiplied by any number, being equal the first power of such number, it follows, that the second, third, fourth, &c. powers of the same number, are produced by multiplying unity one, two, three, four, &c. times by it.

EXAMPLES.

Let 2 be raised to all the successive powers, from the first to the fifth.

$1\times2=2$, 1st power of $2=2^1$.
$1\times2\times2=4$, 2d power of $2=2^2$ its square.
$1\times2\times2\times2=8$, 3d power of $2=2^3$ its cube.
$1\times2\times2\times2\times2=16$, 4th power of $2=2^4$ its biquadrate.
$1\times2\times2\times2\times2\times2=32$, 5th power of $2=2^5$ its sursolid.

PRACTICAL EXAMPLES.

1. A square chamber is 36 feet each way; how many square feet of boards will floor the same?
Ans. 1296.

2. In a plantation 400 perches square, how may square perches? Ans. 160000.

3. If a stone pier was 20 feet every way, how many cubic feet would it contain? Ans. 8000*ft*.

4. Required, the 4th power of 12. Ans. 20736.
5th power of 24. 7962624.
6th power of 48. 12230589464.

From the above examples, we deem the doctrine of raising powers is clear.

EVOLUTION,

OR THE EXTRACTION OF ROOTS.

To extract the square root,

1. Prepare the number for extraction, by pointing it off from units' place, into periods of two figures each.

2. Find, in the table of powers, a square nearest to the first, or left hand period, subtract, bring down the next period, and call this the first dividend.

3. Double the quotient figure for a divisor, and try how often it is contained in all the figures of the dividend, except the one on its right.

4. Place this in the quotient for the second figure of the root, as well as to the right of the divisor.

5. Multiply by this quotient figure, as in division; the product subtract as before, and to the difference bring down the third period.

6. Proceed in like manner, still doubling the quotient figures for a new divisor, and bringing down another period each time for a new dividend, until the whole be brought down, and the thing is done.

EXAMPLES.

1. What is the square root of 10342656?

```
  10‘34‘26‘56(3216
   9
   —
 62)134
    124
    ———
  641)1026
       641
      ————
  6426)38556
       38556
       —————
```

Here, the number consisted of 4 periods; in the table of powers, I found a square nearest to the first period, 10, and set the root, 3, in the quotient; I now doubled 3 for a divisor, and said 6 into 13, the first dividend, all but its right hand figure, and found it was contained in 13 twice; I now placed 2 for the second figure of the root and also beside 6, the divisor, and multiplied 62 by 2, which gave 124; this I subtracted from

So I proceeded until the work was finished.

2. Extract the root of 151321. Ans. 389. Root.
2985984. 1728.
23097636. 4806.
2990667969. 54687.

The use of the Square Root is very great; but I shall content myself with giving a few examples only, in this place.

1. If the General of an army desired to scale the walls of a city, which were 15 feet high, and surrounded by an inaccessible mote, of 20 feet wide, he could, by help of the Square Root, ascertain the length of a ladder which would reach from the outside of the mote to the top of the wall. Thus:
$\sqrt{20 \times 20 + 15 \times 15}$*$= 25$ feet, the length required.

2. If the length of a building be 80 feet, and its breadth 60, what distance should it measure from corner to corner diagonally? Ans. 100 feet.

3. If a person desired to lay out the foundation of a building by lines, so that it be exactly square, measure 6 feet on one line from the corner, and eight upon the other, (crossing the first at said corner;) if these two points be just 10 feet asunder, the lines cross at right angles and are squared with each other; for $\sqrt{6 \times 6 + 8 \times 8} = 10$.

4. The length of one side of a garden was 400 feet, and its breadth, or the length of the other side, 300, and it measured diagonally 500 feet; was i square, or in other words, right-angled?
$(400 \times 400 + 300 \times 300)^{\frac{1}{2}} = 500$; which proves tha the garden was right-angled.

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* By the 47th proposition of the 1st book of Euclid Elements of Geometry, it is proved, that the square ro of the sum of the squares of any two numbers or lines, equal to the root of a number whose square would be eq to the sum of the squares of both the former numbers, a line equal the side of a square as large as both squares made upon the given lines.

To extract the Cube root,

RULE.

1. Prepare the number for extraction, by pointing it off into periods of three figures each, from the place of units.

2. Find a cube in the table of powers, nearest to the first, or left hand period; place its root in the quotient, and subtract the cube itself from the first, or left hand period.

3. To this difference bring down the first, or left hand figure of the second period, and call this the *resolvend.*

4. Multiply the square of the root found by 3 (the index of the cube) for a divisor.

5. Try how often this divisor will go into the resolvend; place the result in the quotient, and cube all the quotient figures for a subtrahend.

6. Draw a line underneath, place down the subtrahend below the two first periods, and subtract.

7. To this difference bring down the left hand figure of the third period, and call this the second resolvend; multiply the square of all the quotient figures now found by 3 (as before) for a second divisor.

8. Try this divisor into the second resolvend, place the result in the quotient, cube all the quotient figures still again for a new subtrahend; subtract, &c. as before, bringing down the left hand figure of each successive period, to the last, dividing and cubing as before. If the given number be a perfect cube, the last subtrahend will be equal to it exactly; if not, then a fractional part belongs to the root.

EXAMPLES.

1. What is the cube root of 34965783?

34‘965‘783(327 Ans.
27
27) 79 $3\times3\times3=27$, first divisor.
32768 $32\times32\times32=32768$, first subtrahend.
3072)21977 $32\times32\times3=3072$, second divisor.
$327\times327\times327=34965783$, the second subtrahend, which happens to be equal to the given number; therefore the thing is done.

Operation. First, I search the table of powers for a cube nearest to 34, and find 27 to be such a one; the root 3 I place in the quotient, and subtract 27 from 34; to the remainder, 7, I bring down 9, the left figure of the second period, and call this the resolvend; next, I multiply the square of the quotient figure by 3 (the index of the power) for a divisor, which I try into 79, the resolvend, and find it goes twice, this 2 I place in the quotient; I now cube both, viz. 32, for a subtrahend, and find it= 32768; this I place below the two first; and so proceed as above, until the whole is performed.

2. What is the cube root of 16194277? Ans. 253.
3. What is the cube root of 84604519? 439.
4. What is the cube root of 43603682428 7?
Ans. 7583.

The cube root is used in the investigation of multitudes of useful problems in mensuration of solids. A few examples of which follow.

1. Required, the length of the side of a cubic vat, capable of containing 847 hogsheads of wine.

Operation. $\sqrt[3]{847\times63\times231}=231$ inches$=$19*f.* 3*in.*

2. Required, the length of the side of a cubic vat, capable of containing 2209 barrels of beer.

Ans. 282*in.* or 23*ft.* 6*in.*

The extraction of roots of higher powers is here omitted, as not being of much use in this place.

SQUARE ROOT,

FURTHER ILLUSTRATED AND EXEMPLIFIED.

To extract the Square Root of fractional expressions (called vulgar,)

RULE.

1. Reduce the fraction (if necessary) to its lowest terms; extract the root of the numerator, and that of the denominator; place the roots of these in the same order, and the new fraction will be the root required.

2. If a mixed number, reduce it to an improper fraction; extract the root of both sides as before, and the thing is done.

CASE 1.

EXAMPLES.

1. If the area of a square surface, or field, be $\frac{25}{36}$ of a square mile, what is the length of one of its sides?

$\sqrt{\frac{25}{36}}=\frac{5}{6}$ of a mile in length.

2. If the superficial content of a square board be $\frac{900}{1024}$ of a square yard, what is the length of one of its sides? Ans. $\frac{30}{32}$ or $\frac{15}{16}=2$*ft.* 9*in.* 9*sec.*

3. If the area of a small garden was $10\frac{39}{49}$ square perches, how many feet long is one of its sides?

Ans. $54\frac{3}{14}$*ft.*

To extract the square roots of surd fractions,*

RULE.

1. If a simple fraction, reduce it to a decimal equal thereto; extract the root of this decimal, and the thing is done.

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* Surds are such powers as have no rational, or even roots.

2. If a mixed number, reduce the fractional part to a decimal, annex it to the whole part; take the root, and the thing is done.

CASE 2.

EXAMPLES.

1. Suppose the surface of the lid of a square snuff-box was just $8\frac{3}{7}$ square inches, how long is it.

First, 7)3.0(.4285+ and 8.4285(2.9031+*in.* *Ans.*

$$\begin{array}{rr}
28 & 4 \\
\hline
20 & 49)442 \\
14 & 441 \\
\hline
60 & 5803)18500 \\
56 & 17409 \\
\hline
40 & 58061)109100 \\
35 & 58061 \\
\hline
5 & 51039
\end{array}$$

Or, $\frac{59}{7}=8.4285+$

And, $\sqrt{8.4285}=2.9031+$ Ans.

NOTE. Both the fraction of this power, and its root, might be carried much farther, if necessary.

2. Suppose the area of a building lot was exactly $76\frac{11}{12}$ square perches, how many yards of paling will inclose it? Ans. 192.09.

CASE 3.

EXAMPLES.

1. What is the mean proportional between 16 and 100? $\sqrt{16\times100}=40$. Ans.

For, $16 : 40 :: 40 : \frac{40\times40}{16}=100$. Proof.

2. Required, a mean proportional between $\frac{225}{16}$ and $\frac{289}{25}$. Ans. 12¾.

Carpenters, in framing timber, use various rules for ascertaining the length of a brace to extend any length upwards, and along a sill the same length, or to extend equal lengths upon any two pieces at right angles to each other; what is the most certain rule amongst them?

3. Suppose it was required to find the length of a brace that would extend 4 feet upon each of two pieces of timber, making a square with each other.

By the 47th proposition of the 1. book of Euclid.

$$(4^2+4^2)^{\frac{1}{2}}=\sqrt{32}=5\mathit{ft.}\ 7\mathit{in.}\ 10\mathit{sec.}\ \text{Ans.}$$

But a practical rule,* that answers very well for business, is, just to consider one of the perpendiculars (as 4) to be so many feet, and the same multiplied by 5, so many inches, which reduced to feet and added to the former feet, gives the length required, a little too long; but is supposed the better for it, on account of the work shrinking as it dries.

EXAMPLE.

Find the length of a brace to extend 6 feet along two pieces of timber, at right angles to each other.

Set down 6. for feet.

$$\text{Then } \frac{6\times5}{12}=2.6\mathit{in.}$$

8.6 length required.

By the 1st rule, $(6^2+6^2)^{\frac{1}{2}}=8.48+$ or 8*ft.*5*in.*9*sec* within less than $\frac{1}{4}$ of an inch of what the second rule brings out.

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* This practical rule arises from the circumstance of the legs of the right angled triangle being equal, and the ratio of the hypothenuse to the legs. The true number to multiply by, is easily deduced from the same principle; it will be 4.96+ in all cases, whether the equal legs be longer or shorter.

4. There are two columns, in the ruins of Persepolis, left standing upright; one is 64 feet above the plain, the other 50. Between these, in a right line, stands an ancient statue; the head whereof is 97 feet from the summit of the higher, and 86 feet from the top of the lower column; the base whereof measures just 76 feet to the centre of the figure's base. Required the distance between the tops of the columns.

Ans. 169.95839 feet, when the top of the statue is higher than the columns; but if lower, then 157.03+ = distance between their tops.

5. There are points of the Andes, in South America, that are 4 miles above the level of the sea; to what distance could a person see from the top of such an elevated point, provided the atmosphere was perfectly clear, and not assisted by refraction.

If we put the earth's semi-diameter at 4000 miles, then $\sqrt{4004 \times 4004 - 4000 \times 4000} = 178.93+$ miles, the distance required, which is but about twice as far as a person could see from a point of land elevated but one mile above the level of the sea.

RULE,

For finding the distance that an object may be seen at sea, elevated any height above the level of the water.

Add to the earth's semi-diameter, in feet, the height of the object; square the sum; next square the number of feet in the earth's diameter; take the difference of these squares; then (by E. 1. 47.) the square root of the difference of those squares will be the distance required, in feet.

Note. In all cases where the square root of the sum, or difference of the squares, of any two numbers is required, the principle is inferred from the proposition in Euclid's Elements of Geometry, above quoted.

6. The sum of any two numbers multiplied by their difference, plus the square of the lesser, is equal the square of the greater. And the sum of any two numbers, multiplied by their difference, subtracted from the square of the greater, will leave the square of the lesser. Required, a demonstration.

CUBE ROOT,

FURTHER EXEMPLIFIED.

To extract the cube root of a vulgar fraction,

RULE.

1. Reduce the given fraction, if necessary, to its lowest terms; extract the root of both sides, place it in the same order, and the thing is done.

2. If the quantity be mixed, reduce it to an improper fraction; extract as before, and the thing is done.

3. If surds, reduce them to their equivalent decimal values; extract the root of such decimal, if alone, if joined to a whole number, then the root of both when joined together, and the thing is accomplished.

EXAMPLES.

1. The solid content of a cubic box is $\frac{324}{1500}$ of a cubic yard; what is the length of one of its sides?

If the fraction be $\div$ by 12 or 3)$\frac{324}{1500}$(=$\frac{108}{500}$ and 4)$\frac{108}{500}=\frac{27}{125}$, wherefore $\sqrt[3]{\frac{27}{125}}=\frac{3}{5}$ of a yard long= 1*ft.* 9.6*in.* Ans.

2. The solid content of a cubic vat is $5\frac{104}{125}$ cubic yards; required, the length, breadth and depth.

Ans. 5.4 feet.

3. The solid content of a cubic vat is $7\frac{6}{7}$ cubic yards; what is the length of its sides?

$7\frac{6}{7}=\frac{55}{7}=7.857142+$ and $(7.857142)^{\frac{1}{3}}=$
1.97+*yds.* or 5.91*ft.* Ans.

4. The solid content of a cellar, which is alike long, broad and deep, is 100¾ cubic yards; required, the length of its sides. Ans. 13.95 + *ft*.

To find the side of a cube that shall be equal in solidity to any given solid, as a globe, cylinder, prism, cone, &c.

RULE.

Extract the cube root of the solid content of the given body, which root will be the side of the cube required.

EXAMPLE.

1. What is the whole superficies of a cubic block which contains exactly 300 solid feet?

Ans. 268.456326 +

Having the dimensions of any solid body, to find the dimensions of another similar solid, that shall be any number of times greater or less than the solid given,

RULE.

Multiply the cube of each side by the difference* between the solid given and that required, if greater (or divide by the difference, if less) than the solid given; then extract the cube root of each product or quotient, which will give the dimensions of the solid required.†

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* What is meant here by the difference, is, the number of times the required solid is to be greater or less than the given one.

† DEMONSTRATION. Let us assume a solid, the sides of which may be 2, 3, and 4 feet long, respectively; its solidity in cubic feet will be $2 \times 3 \times 4 = 24$ feet. Now,

EXAMPLES.

1. The dimensions of a ship are, $\left\{\begin{matrix}\text{Keel,} & 125\textit{ft. long.}\\ \text{Beam,} & 25\\ \text{Depth of hold} & 15\end{matrix}\right.$

What dimensions should a ship of similar form have, to contain, or carry three times the burthen?

Ans. $\left\{\begin{matrix}\text{Length of keel} & 180.28\textit{ft.}\\ \text{Breadth of beam} & 36.05\\ \text{Depth of hold} & 21.63\end{matrix}\right.$

2. Find the dimensions of a similar ship, that shall contain, or carry, just half the burthen of that whose dimensions are given.

Ans. $\left\{\begin{matrix}\text{Length of keel} & 99.21\textit{ft.}\\ \text{beam} & 19.84\\ \text{hold} & 11.09\end{matrix}\right.$

Like solids are in triplicate proportion to their homologous (or like) sides; therefore, it will be, as the cube of a dimension : is to its given weight : : so is the cube of any like dimension : to the weight sought. [Inferred from proposition 8. book 12. Euclid.]

---

suppose we would know the dimensions of a similar solid, just twice as large; it is had per rule.

$$\left.\begin{matrix}2\times2\times2\\3\times3\times3\\4\times4\times4\end{matrix}\right\}\times2\left\{\begin{matrix}=16\\=54\\=128\end{matrix}\right\}\text{and}\left\{\begin{matrix}\sqrt[3]{16}=2.519+\\\sqrt[3]{54}=3.779+\\\sqrt[3]{128}=5.037+\end{matrix}\right.$$

The dimensions required; for $\sqrt[3]{16}\times\sqrt[3]{54}\times\sqrt[3]{128}=\sqrt[3]{110592}=48=$ twice the given solid, (which we assumed,) as was required. If the given solid was a cube, it is only necessary (in case we want to find a similar one, two or more times as large) to multiply the cube of the side (which is the content) by 2 or more, as the case may be; the cube root of which product will be the side of another 2, 3, or more times as large, &c.

EXAMPLES.

1. If a ship of 400 tons burthen, be 80 feet long in the keel; what is the burthen of another ship, the keel of which is 100 feet long. Ans. 781*T.* 5*C.*

2. Suppose a cannon ball, of 4 inches diameter, weighs 18*lb.*; what is the diameter of another, that weighs 42*lb.*? Ans. 5.30+

3. Suppose a mortar shell, of 8 inches diameter, weighs 50*lb.*; what is the diameter of a shell that weighs 100*lb.*? Ans. 10.08+ inches.

To find two mean proportionals between two given numbers,

RULE.

Multiply the square of the lesser extreme, by the greater extreme, the cube root of the product will be the lesser mean; again, multiply the square of the greater extreme, by the lesser extreme, the cube root of the product will be the greater mean.*

---

* Demonstration. Let us assume any two numbers, as 2 and 16, and find two mean proportionals between them; let these proportionals be denoted by x and y; then by the nature of geometrical ratio, $2 : x :: y : 16$. Now, because the second term, x, is a mean proportional between 2 and y, and y a mean proportional between x and 16, $\sqrt{16x}=y$; for the square root of the product of any two numbers is a mean proportional between those numbers; hence then, $2 : x :: \sqrt{16x} : 16$. Now, by multiplying means and extremes, $2 \times 16 = x \times \sqrt[3]{16x}$; that is, $32 = \sqrt{16x^3}$; now by squaring both sides, and by division, $x^3 = \frac{1024}{16} = 64$, and by extraction, $x = \sqrt[3]{64} = 4$ the lesser mean, and $(16 \times 16 \times 2)^{\frac{1}{3}} = 8$, the greater mean, which operation, written out in words, is the rule above.

EXAMPLES.

1. Required, two mean proportionals between 1 and 2. $1 \times 1 \times 2 = 2$ and $\sqrt[3]{2}$ = lesser mean.

Again, $2 \times 2 \times 1 = 4$ and $\sqrt[3]{4}$ = greater mean.

For, $1 : \sqrt[3]{2} :: \sqrt[3]{4} : 2$. Now multiply means and extremes, $1 \times 2 = \sqrt[3]{2} \times \sqrt[3]{4} = \sqrt[3]{8} = 2$. Proof.

2. Required, two mean proportionals between 4 and 256. Ans. 16 and 64.

3. Required, two mean proportionals between 3 and 11. Ans. $\sqrt[3]{99}$ and $\sqrt[3]{363}$.

NOTE. More examples of the uses of the Cube Root might be shewn; but I presume I have given sufficient to make the subject familiar enough to any one, to extend it to any length farther that may be thought necessary.

A GENERAL RULE FOR EXTRACTING THE ROOTS OF ALL POWERS.

1. Prepare the given number for extraction, by pointing off from the place of unity, as the root required directs. If the square root, the number of figures to a period must be two; if the cube root, three; the biquadrate root, four, &c. because the indices of the powers are, 2, 3, 4, &c.

2. Find the first figure in the root by your own judgment, or by inspection into the table of powers.

3. Subtract it from the given number.

4. Augment the remainder by the left hand figure of the second period.

5. Involve the whole root, last found, into the next inferior power to that which is given.

6. Multiply it by the index of the given power, and call this your divisor.

7. Find a quotient figure by common division, and annex it to the root.

8. Involve all the root thus found, into the given power.

9. Subtract this power (always) from as many points of the given power as you have brought down, beginning at the lowest place.

10. To the remainder bring down the first figure of the next point, for a new dividend.

11. Find a new divisor, as before; and in like manner proceed till the work is ended.

EXAMPLE.

Let the square cube root of 782757789696 be extracted.

782757'789696(96
9×9×9×9×9×9= 531441
9×9×9×9×9×6=354294)2513161
96×96×96×96×96×96=782757789696
0

This, you see, is done precisely as the square or cube roots, and so is every higher power.

There are many methods given for extracting the roots of powers, by approximation, &c.; but the above general rule is as good as any that I know at present, and may suffice.

INSURANCE.

A *writ*, or *policy*, of *Insurance*, is a security passed from the *insurer* to the insured; whereby the insurers, or underwriters, in consideration of a certain premium, are bound to make good to the insured all such property mentioned therein, or an equivalent thereto, against all hazards, perils, losses, &c. or against some particular event.

The *policy*, when made, becomes the property of the insured. It is executed by the insurer or insurers, by subscribing his or their names to it; from which they are called underwriters.

The *premium* is rated at so much per cent, on the property insured, and rises or falls, according to the risk, or hazard run.

In cases of total loss, the underwriter, or underwriters, make good to the insured, the amount insured; deducting 2 per cent, which is called discount.

Abandonment, is that right which the insured has, in case of partial loss, to relinquish his interests in the property insured, to the underwriter, or underwriters, and claim the amount insured, as if for a total loss.

Salvage, is an allowance, or premium, made by the owners of ship and cargo, for saving the ship and cargo, or both, from the dangers of the sea, pirates, or enemies; and, till such premium is made, the person saving the property is justifiable in retaining it.

General average, is a contribution proportionably borne by all concerned in ship and cargo, where a partial loss is incurred for the preservation of the rest; though it is in the end made good, by the underwriters, in proportion to the sums they have underwritten.

Partial loss, signifies the damage a ship or goods may have sustained, in the voyage, from some of the perils insured against.

Average loss, must be sustained by the owners of ship and cargo, or freight, in proportion to their respective adventures at hazard. The average loss was formerly put at 10 per cent; but it must now amount to 5 per cent.

CASE 1.

RULE.

Insurance is calculated as simple interest for years.

EXAMPLES.

1. What is the insurance on \$11000 at 3 per cent?
$11000 \times .03 = 330$. Ans.

2. What is the insurance on \$17500 at $2\frac{1}{2}$ per cent?
Ans. \$427.50.

Adventurers generally take out a policy for such amount, as will be equal to the sum at hazard, *together* with the insurance.*

RULE.

Multiply the given invoice by 100, and divide by 100 minus the premium, and the quotient will shew the amount that covers it.†

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* The *premium* is generally received by the insurer at the time the policy is given to the insured; but must be included in the policy with the amount of the adventure insured. It is esteemed the most material part of a policy, on account of its being the *consideration on* which the contract is founded. The insurer acknowledging the receipts of the premium, by signing the policy, makes himself liable for the losses that may happen.

NOTE. A policy is only signed by the insurer, or insurers, and not by the insured.

† To find the sum for which a policy should be taken out, to cover a given sum.

Let $h=100$, $p=$premium, $a=$amount to be insured upon, and $s=$sum to be covered; then

$$h-p : h :: s : \frac{sh}{h-p}=a.$$

When a policy is taken out for a certain sum, in order to cover a given sum,

$$a : s :: h : \frac{sh}{a}=h-p, \text{ or } h-\frac{sh}{a}=p.$$

When the policy for covering any sum, and the premium per cent, are given, to find the sum to be covered,

$$h : h-p :: a : \frac{ah-ap}{h}=s.$$

EXAMPLES.

1. For what amount must I take out a policy, to cover an invoice of $9000 at 9 per cent?

$\overline{100-9} : 100 :: 9000 : 9890.10.9+$ Ans.

2. For what amount must I take out a policy, to cover an invoice of $37004, at 5 per cent?

Ans. 38951.57.8+.

CASE 2.

When the amount of the policy and premium are given, to find the sum covered, multiply the amount

---

When a given sum is adventured several voyages round, from one place to another, either at the same or at different risks, from place to place, and it is required to take out a policy for such a sum as will cover the adventure all round, supposing the risk out and home to be equal and tantamount to the several given risks,

For the first voyage. Second voyage.

$$h-p : h :: s : \frac{sh}{h-p}=a. \qquad h-p : h :: \frac{sh}{h-p} : \frac{sh^2}{(h-p)^2}=a.$$

$$\frac{a\times(h-p)}{h}=s. \qquad \frac{a\times(h-p)^2}{h^2}=s.$$

$$h-\frac{sh}{a}=p. \qquad h-\frac{\sqrt{h^2s}}{a}=p.$$

Third voyoge.

$$h-p : h :: \frac{sh^2}{\overline{h-p\,}\rceil^{2}} : \frac{sh^3}{\overline{h-p\,}\rceil^{3}}=a. \quad \frac{a\times\overline{h-p\,}\rceil^{3}}{h^3}=s.$$

$$h-\frac{\sqrt[3]{h^3s}}{a}=p.$$

And so on for any number of voyages. If m=exponent of any power, then

$$\frac{sh^m}{\overline{h-p}\times\overline{h-p}\times\overline{h-p},\ \&c.}$$ = sum to be insured upon all round; and $h-\frac{\sqrt[m]{h^m\times s}}{a}$ = premium all round, tantamount to the several given premiums.

to be insured by 100 minus the premium, divide by 100, and the quotient will shew the sum to be covered.

EXAMPLES.

1. A merchant insuring an adventure, at 7 per cent, took out a policy for $8000; what was the sum at risk?

$$\frac{8000\times\overline{100-7}}{100}=\$7440.\ \text{Ans.}$$

2. A merchant insuring an adventure, at 9 per cent, took out a policy for $9000; what was the sum at risk? Ans. $8190.

CASE 3.

When the amount and sum covered are given, to find the premium and rebate, multiply the sum to be covered by 100, divide by the amount to be *insured*, and the quotient will be 100—premium *plus the rebate*; from which the premium is easily had, by subtracting said quotient from 100.

EXAMPLES.

1. A merchant insuring $17000, took out a policy for $18000; what was the premium and rebate?

$$\frac{17000\times100}{18000}=94\tfrac{4}{9}\text{ and }100-94\tfrac{4}{9}=5\tfrac{5}{9}\ \text{Ans.}$$

2 rebate.

—

$3\frac{5}{9}$ premium.

2. A merchant insuring $37404, took out a policy for $40000; required, the premium and rebate.

Ans. $6\frac{49}{100}$

2 rebate.

—

$4\frac{49}{100}$ premium

CASE 4.

When the adventure is continued from one part to another, at the same or different risks, to find the amount that will cover the adventure all round, subtract each premium from 100, and multiply the several remainders together for a divisor, raise 100 to a power whose index is equal the number of risks, multiply this by the sum adventured; the product, divided by said divisor, will be the amount of the policy to be insured all round.*

EXAMPLES.

1. A merchant insures an adventure of $3000 to Savanna, at 5 per cent, thence to Kingston, Jamaica, at 3 per cent, thence to Philadelphia, at 2 per cent; for how much must he take out a policy to cover the adventure round?

$\overline{100-5}\times\overline{100-3}\times\overline{100-2} : 100\times100\times100 :: 3000 :$ Ans.

2. A merchant adventured $9000 from Newburyport to Savanna, from thence to Barbadoes, from thence to Bermuda, and from thence home, at 6½ per cent from port to port; for what sum must I take

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* The reason of this rule is founded upon the insured allowing the insurers an additional premium, on account of the increase of risk; which additional premium is ascertained for the first port of destination, as in case 1, by saying, as 100 less the premium per cent, is to 100, so is the amount of the adventure, to the sum to be covered by the policy; which sum is made the adventure to the 2d port of destination, and on which the premium for the 2d is calculated, and so on, for as many as the number of given risks. I do not, however, see the propriety of such a rule in mercantile arithmetic; it being too incommodious for practice; when it might be easily remedied, by adding simply an equivalent to the premium first named; for instance, making 5 per cent, 5½, or 5¾, &c. according to the increase of risk.

out a policy to cover his adventure the voyage round, supposing the risk to be equal out and home, and tantamount to the several given risks? Ans.

CASE 5.

When the adventure is at different risks round, and a premium required equal to a mean value, or tantamount to the several rates per cent; we *must* find the amount to be insured, by the last rule in case 4; subtract the sum adventured from the amount, the remainder is the entire of the discount and premium. Then multiply said difference by 100, divide by the amount, and the quotient will be the rate per cent; from which subtract the discount, the remainder is the premium which must be paid to the underwriters.

EXAMPLE.

A. G. in Philadelphia, insures $10119.33 to Savanna, at 2½ per cent; thence to New Orleans, at 7; thence to Gaudaloupe, at 4; and from thence home, at 5; what amount will cover this adventure round, with 2 per cent, in case of loss?

First the amount is found by case 4. =	$12500
Sum adventured,	10119.33
Insurance and discount,	2380.67

Then, $\frac{2380.67 \times 100}{12500} = 19.065 =$ premium & rebate.
2 deduct rebate.

17.045 rate per ct. or prem.

$$\left.\begin{array}{l}\text{Again, } 12500 \text{ at } 17.045 \text{ per ct.} = 2130.625 \\ \quad 12500 \text{ at } 2 \text{ per cent,} \quad = \quad 250\end{array}\right\} \text{sum} = 2380.625.$$

Wherefore, 12500—2380.625 = 10119.375.

CASE 6.

To find the equal premium paid on a policy taken out to cover an adventure to two or more places, we must multiply the adventure 100 as often as there are risks, and divide the product by the amount for which the policy is drawn; next extract that root of the quotient denoted by the number of risks; this root taken from 100, will give the equal rate or premium.

EXAMPLES.

1. A merchant in Philadelphia, adventures $9000 to London, from thence to Jamaica, and from thence home; to cover which, he took out a policy for $10000; what was the rate per cent premium?

$$100-\left(\frac{9000\times100^3}{10000}\right)^{\frac{1}{3}}=$$

2. A merchant in Philadelphia, adventures $75000 to Liverpool, from thence to the island of Trinidad, and from thence home; to cover which he took out a policy for $80000; what was the rate per cent premium? Ans.

CASE 7.

When an adventure is insured out and home, at one risk, at a given rate per cent, and the voyage terminates short of what was at first intended; to find what the underwriter must receive per cent,

1. If just half the voyage is performed, it must be considered as two equal risks; if one-third, then as three equal risks; if one fourth, four equal risks, &c.; and by case 4 must be found the amount which will cover the adventure round.

2. Involve 100 to that power denoted by the number of risks, and multiply this power by the sum adventured.

3. Divide this product by the aforesaid amount.

4. Extract that root of the quotient denoted by the number of risks.

5. Take this root from 100, the remainder *will be* the sum per cent, which the underwriter must receive.

EXAMPLE.

A merchant covers $7404, at 6 per cent, *from* Philadelphia to Kingston, Jamaica, and home again; but the voyage terminating at Kingston, what must the insurer receive per cent.

$$\overline{100-6} : 100 :: 7504 : 7865.74.4+$$

$$\frac{100\times100\times7404}{7865.74.4+}=9412.713+$$

Again, $100-\sqrt{9412.713}=2.98$ per cent, to be paid the insurer on the above amount.

GENERAL AVERAGE.

Whatever the master of a ship in distress, with the advice of his officers and sailors, deliberately resolves to do, for the preservation of the whole, in cutting away masts or cables, or in throwing goods overboard, to lighten the vessel, which is what is meant by jetson, is in all places permitted to be brought into a general average; in which, all who are concerned in ship, freight and cargo, are to bear an equal or proportionable part of the loss of what was so sacrificed for the common welfare, and it must be made good by the insurers, such proportions as they have underwritten.

EXAMPLES OF ADJUSTED AVERAGES.

1. A loaded ship met with such bad weather, that the master and mariners found it impossible to save her, without throwing part of her cargo overboard,

which they are authorised to do for preservation. Being thus necessitated, they threw such goods as lay nearest at hand, and lightened the ship of 10 casks of hardware, and 40 pipes of Madeira wine; which they judged sufficient to keep her from sinking. Soon after that, the ship arrived at her destined port, and then an average bill was immediately made, in order to adjust the loss, and to pay the proprietors of those goods, which were thrown overboard for the good of the whole.

Average accrued to ship Seahorse, for goods thrown overboard, for preservation of ship, freight, and cargo.

Ship valued at - - - -	$14000.
Freight (wages and victuals deducted)	3000.
Andrew Jones's value of goods, -	5000.
Philip Mayers's value of goods, -	2700.
Robert Johnston's value of goods, -	8300.
Joseph Baumgarden for 40 pipes of wine,	4000.
Daniel Duffy for 10 casks of hardware,	5000.
	42000.

Mr. Joseph Baumgarden's wine thrown overboard, valued at	$4000.	9000.
Daniel Duffy's hardware,	5000.	

Now let $\frac{9000 \times 100}{42000} = 21\frac{3}{7}$ per cent.

The ship must pay to Joseph Baumgarden and Daniel Duffy, for $14000 at $21\frac{3}{7}$ per cent,	$3000.
The freight $3000 at the same rate,	$642.85\frac{5}{7}$
Andrew Jones for $5000 at same,	$1071.42\frac{6}{7}$
Philip Mayer for $2700 at same,	$578.57\frac{1}{7}$
Robert Johnston for $8300 at same rate,	$1778.57\frac{1}{7}$
J. B. & D. D. receive of the owners of the goods saved and the ship's owners	$7071.43\frac{6}{7}$
Their property being insured, the underwriters pay them	$1928.56\frac{1}{7}$
	9000.

The ship Seahorse, captain Dix, laden with hemp, cordage, and iron, bound from Riga to Boston, ran on shore, coming through the grounds at Elsineur. The captain hired a great number of men, and several lighters, to lighten the ship, and to get her afloat again, which was done; but he was obliged to pay $409.23 for their assistance. This expense being incurred to preserve both ship and cargo, the average must consequently be general. When the ship arrived at Boston, the captain immediately made a protest and an average bill which was thus stated:

Average accruing to the ship Seahorse, from Riga to Boston, 1799, for assistance in getting off the strand of Elsineur.

For sundry charges paid at the sound for lighters & assistance in getting off the ship,	$409.23
Protest and postage,	35.37
	444.60

The ship's freight money - -		3460.
Wages for all the people (4*M.* 20*D.*)	560	860.
Victuals for ditto		
		2600.

Ship Seahorse valued at - -	$12000.
Freight valued at - - -	2600.
William Jenkins, for value of hemp,	18000.
Daniel Jones, for value of cordage,	4000.
Enoch Flinn, for value of iron, -	2400.
	39000.

If 39000 lose 444.60, what will 100 lose? Ans. 1.14

The ship then must bear for 12000 at 114*cts.* per 100,	$136.80
The freight amounting to 2600 at same,	29.64
William Jenkins for 18000 at same,	205.20
Daniel Jones for 1000 at same,	45.60
Enoch Flinn for 2400 at same,	27.36
Proof	444.60

Note. The doctrine of Insurance is extensive; if we were to arrange the different heads, it would be necessary to consider each separately.

As 1st. *Of Marine Insurance;* considering,

1. The policy, its nature. 2. The construction to be put on it. 3. Warranties in policies. 4. Proceedings on policies. 5. Of re-assurances and double assurances.

2d. *Of losses under such policies.*

1. Of total losses, by peril of the sea. 2. By capture. 3. Detention. 4. Barratry. 5. Stranding. 6. General, or gross average. 7. Partial loss and adjustment. 8. Salvage and abandonment.

3d. *Of fraud, illegality, or irregularity;* which either vitiate the policy, or prevent a recovery, though a loss happen.

1. Of direct fraud in policies. 2. Of changing the ship. 3. Deviation on the voyage. 4. Sea worthiness. 5. Of wager policies. 6. Of valued policies. 7. Of illegal voyages. 8. Of enemies' ships, &c. 9. Of prohibited goods and commerce. 10. Return of premium, in cases of void or fraudulent policies, &c.

4th. *Of bottomry and respendentia.*
5th. *Of insurance on lives.*
6th. *Of insurance against fire,* &c.

BARTER.

Barter is the exchanging of one commodity for another; and teaches traders to proportion their quantities without loss.

I shall give a few examples in Barter, with the method of working them, as questions in Barter cannot be conveniently classed in separate cases.

EXAMPLES.

1. How many tons of bar iron, at $7.50 per *Cwt.* should I give for 5000 gallons of apple brandy, at 60*cts*. per gallon?

6|0)7.5|0

12.5)5000(400*Cwt.*=20*T*. Ans.

2. How much iron, at $120 per ton, must be given for 100 barrels of flour, at $8.50 per barrel?

$$\frac{8.50\times100}{120}=7T.\ 1C.\ 2q.\ 18\tfrac{2}{3}lb.$$

3. How much tea, at $1.37½ per *lb.*, must be given for 200 gallons of Madeira wine, at $1.62½ per gallon? The given prices being as 11 to 13,*

$$\frac{200\times13}{11}=236\tfrac{4}{11}lb.\ \text{Ans.}$$

4. How much sherry wine, at 87½*cts.* per gallon, must I give for 750 gallons of Lisbon, at 37½*cts.* per gallon? The given prices being as 7 to 3,

$$\frac{750\times3}{7}=321\tfrac{3}{7}\ \text{Ans.}$$

5. How much wheat, at $1.50 per bushel, must I exchange for 1000 bushels of Indian corn, at 75*cts.* per bushel? The prices being as 2 to 1,

$$\frac{1000}{2}=500\ \text{Ans.}$$

6. How much calico, at 31¼*cts.* per yard, must I give for 5000 shingles, at $5.75 per thousand?

$$\frac{5.75\times5}{.31.25}=92yds.\ \text{Ans.}$$

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* In Barter, where the prices are given in dollars and parts of a dollar, or in even parts of a dollar, the ratio to each other is easily discovered by inspection; here 11 is to 13, arises from 1.37½, containing $\frac{11}{8}$ of a dollar, and 1.62½, $\frac{13}{8}$.

7. How much broadcloth, at $6 per yard, cash, but $6.50 credit price, must I give for 300 gallons of peach brandy, at 80*cts.* per gallon, barter price?

$6.50 : 6 :: .80 : 73\frac{1}{13}$ = cash price for a gallon.

And $73\frac{1}{13}=\frac{960}{13}\times300\div6=36$*yds.* 3*q.* 2¾*n.* Ans.

8. How much linen, at 87½*cts.* per yard, barter price, should I give in exchange for 3000*lb.* soal leather, at 25*cts.* ser *lb.* cash, when the barter advance is agreed at 12 per cent? Ans. 960*yds.*

* * *

LOSS AND GAIN,

OR PROFIT AND LOSS.

Profit and Loss is the name of a species of account, by which the profit or loss, on the sale of any commodity, may be known.

EXAMPLES.

1. Bought 300 gallons of whiskey, for 30*cts.* per gallon, and sold it for an advance of 3*cts.* per gallon; required, the whole gain.

$300\times.03=9.$ Ans.

2. Sold 1000 gallons of peach brandy, at 87½ *cts.* per gallon, which was at 20 per cent profit; required, the prime cost per gallon, and the whole profit?

$\underline{120 : 100 :: 87.5 : 72\frac{11}{12}}$ prime cost.

And $\overline{.87.5-.72\frac{11}{12}}\times1000=\$145.83\frac{1}{3}$ gain. Ans.

3. If I buy 1500 bushels of wheat, for $1.37½ per bushel, and sell the whole for $2475, what is the whole gain, and gain per cent?

$1.37\frac{1}{2}\times1400=2062.50$ and $2475-2062.50=$ 412.50 whole gain.

Then, $\frac{412.50\times100}{2062.50}=20$, the rate per cent.

4. If I sell linen for 75*cts.* per yard, and gain $12\frac{1}{2}$ per cent; what would the prime cost of 30 pieces, each $33\frac{1}{3}$ yards, stand me in the whole?

$$\frac{.75\times100}{100+12.5}=.66\tfrac{2}{3}\textit{cts.}\text{ per yard prime cost.}$$

And $33\frac{1}{3}\times30\times.66\frac{2}{3}=\$670$ cost in the whole.

5. Bought a quantity of leather, at 22*cts.* per *lb.*; how should I sell it, to gain 10 per cent profit?

$$\frac{100+10\times22}{100}=24\textit{cts. }2\textit{m.}\text{ Ans.}$$

Philadelphia, June 1, 1820.

6. Shipped on board the Leopard, for Barbadoes, 700 barrels of flour, at $9 per barrel; paid porterage, 6*cts.* per barrel, storage, $7\frac{1}{2}$*cts.*, insurance on the whole, at 5 per cent; how do they stand me on board, taken altogether? Ans. $6709.50

7. Imported from Amsterdam, 20 casks of gin, containing 2080 gallons, at 3 guilders, or 111*cts.*, per gallon; paid here freight $200, porterage $13.50, gauging, 25*cts.* per cask, insurance on *the invoice*, 10 per cent, leakage in the whole, 19 gallons, storage, 27*cts.* per cask. How will I stand, if I should sell it for $1.30 per gallon? Ans. $80.67 loss.

8. Bought goods to the amount of $1400 cash, and sold them at a credit of 4 months, for $1600; what is the gain per cent per annum?

1600—1400=200 gain in 4 months.

And 200×3=600 gain in 12 months.

$$\text{Therefore, }\frac{600\times100}{1400}=42\tfrac{6}{7}\text{ Ans.}$$

9. Bought for cash, $1740 worth of ironmongery, and sold the lot again for $2000, on a credit of 3 months; what is the gain per cent per annum?

Ans. $59\frac{67}{87}$.

10. Bought 3000 gallons of rye whiskey, at a credit of 2 months, for 30*cts.* per gallon, and sold it again at 35*cts.* per gallon, on a credit of 3 months; required, the gain ger cent. Ans. 64.36+

11. Bought 12 hogsheads of apple brandy, each 112 gallons, at 40*cts.* per gallon. One hogshead by accident stove; it is required to sell the remainder so that the owner may afford a credit of 2 months, and clear by the bargain 10 per cent.

Ans. 46.92*cts.* per gallon.

12. A distiller is about purchasing 10000 gallons of molasses, which he can have at 50*cts.* per gallon, cash, or at 52*cts.* with 2 months credit; it is required to know, if he would have any advantage in borrowing money at 8 per cent, to take the molasses at cash price.

The cash price of the molasses is $5000, credit price 5200; present worth of credit price 5131.578+ amount of cash price at 8 per cent $$5066\frac{2}{3}$, and 5131.578—$5066\frac{2}{3}$=64.912+ advantage in favor of borrowing.

13. Shipped for Rotterdam 500 salt ox hides, weight nett 400*C.* 2*q.* 14*lb.* at $5 per *Cwt.*; paid duty, charges, &c. $60; the hides weighed in Rotterdam, 39370*lb.*; my correspondent sells them at 15 guilders, or $5.55 for 100*lb.*; he deducts for weight, charges, &c. 650 guilders, or $240.50, and for his commission 2 per cent; the nett proceeds he remits to London, at 2*s.* sterling, or $44\frac{4}{9}$*cts.* per guilder, for which I draw at 2 per cent above par. What do I gain by the adventure? Ans. $$265.82\frac{1}{2}$ gain.

14. Borrowed $5000, for which I pay 6 per cent interest; with which sum I purchased goods at 6 months credit, but was allowed discount at the same rate per cent, for paying cash; sold said goods at 3 per cent advance, but gave a credit of 2 months. What is gained or lost by the trade?

Ans. $132.02.8 gain.

NOTE. Questions, such as the above, should be carefully studied; being of material consequence in trade. The nature of such reckonings being well understood, may save many a blunder in settlements, and be profitable to the merchant.

PARTNERSHIP OR COMPANY,

THE SAME AS FELLOWSHIP.

Company, or fellowship, in an arithmetical *sense*, is a rule by which merchants in partnership, *adjust* their accounts, in proportion to stock and time.

The gain or loss, with the several sums at hazard, given to find the proportion each partner is to have.

CASE 1.

RULE.

Add the several stocks into one sum, multiply the entire gain or loss by each particular share of stock, divide by the aggregate, or sum of said stocks, and the thing is done.

EXAMPLES.

1. A and B entered into partnership; A put in 2000 dollars, and B put in 1500 dollars. They, in a certain time, gain 800 dollars, which they agree to draw from stock; how should they divide it?

$$\text{Ans. A's share}=457\tfrac{1}{7} \quad \text{B's share}=342\tfrac{6}{7} \Big\} \$800.$$

2. A and B enter into partnership; A has in goods at cash price, 3400 dollars worth, and cash 1300 dollars; B puts in 1200 dollars, and agrees to pay for A, a debt of 1100 dollars, for which A gives B a title to that amount of his goods. Now, suppose A agrees to take B's note for what B's funds want of being equal to his own (say the note bears legal interest, and is not reckoned in the partnership,) what amount should the note be drawn for to make them equal?

Amount of A's goods, 3400
cash, 1300

4700
A sells to B goods for 1100

3600
B's cash 1200 } 2300
goods 1100

Difference 1300 For half this sum, viz. 650 dollars, B should give his note. A's stock in trade would then be

3600—650=2950

$$\left.\begin{array}{l}\text{And B's equi-}\\ \text{valent compos-}\\ \text{ed of}\end{array}\right.\left\{\begin{array}{ll}\text{goods,} & 1100\\ \text{cash,} & 1200\\ \text{note,} & 650\end{array}\right\}=2950$$

This note pays for so much of A's goods.

The above might be readily considered wrong; but a fair statement, according to the true method of accounts, will prove it right beyond dispute. I would be permitted further to remark, that many partners, on the one hand, gain more on a dissolution of trade than their due, and on the other, lose heavily, without knowing how.

CASE 3.

In apportioning the effects of bankrupts amongst their creditors, it is more convenient to find the proportion for one dollar, &c.; which will be a constant multiplier for each debt.

EXAMPLES.

1. A merchant being in trade, delivers up to his creditors, in cash, 1000 dollars, goods, 1500 dollars, book debts, 3000 dollars, notes and bonds, 700 dollars; he owes to John Jenks 1000 dollars, to David Owen, 1100 dollars, to Caleb Strong, 3700 dollars, and to Henry Jones, 2700 dollars.

The merchant has in		Owes,	
Cash,	$1000	To Jenks,	$1000
Goods,	1500	Owen,	1100
Book debts,	3000	Strong,	3700
Notes & bonds,	700	Jones,	2700
Whole effects,	6200	Whole debt,	8500

Then per case 1. 8500 : 6200 :: 1 : $\frac{62}{85}$ of a dollar.

$$\text{And}\begin{cases} 1000\times\frac{62}{85}=\text{Jenks' share}= \\ 1100\times\frac{62}{85}=\text{Owen's} \quad = \\ 3700\times\frac{62}{85}=\text{Strong's} \quad = \\ 2700\times\frac{62}{85}=\text{Jones'} \quad = \end{cases}$$

2. A owes to B 750 dollars, to C 1100, to D 1350, to E 1975, and to F 1650; and has in goods, cash, and other effects, only to the value of 2000 dollars; what sum should each creditor receive?

By working as above, the value of a dollar in each debt will be found $=\frac{200}{687}$; which multiplied into each will give the respective answers.

CASE 3.

When stocks have been put in trade for different lengths of time, and settled with regard both to stock and time, we must multiply each man's stock and time, and divide by said divisor, and the thing is done.*

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* When the times are equal, the shares of gain or loss are evidently in proportion to the respective stocks; and when the stocks are equal, the shares are in proportion to the times; but when stocks and times are both unequal

EXAMPLES.

1. Two merchants made a company; A put in 200 dollars for 5 months, and B 150 dollars for 6 months, and they gain 70 dollars; what share of the profit belongs to each?

$$200\times5=1000. \qquad 150\times6=900.$$

$$\text{And } \overline{1000+900} : 1000 :: 70 : 36\tfrac{16}{19} \text{ A's share.}$$

$$\overline{1000+900} : 900 :: 70 : 33\tfrac{3}{19} \text{ B's share.}$$

2. Three merchants join trade; A puts in 500 dollars for 3 months, B 600 for 4 months, and C 700 for 5 months; what share of 200 dollars gain should each have?

$$\text{Ans.} \begin{cases} \text{A's } 40\tfrac{20}{37} \\ \text{B's } 64\tfrac{32}{37} \\ \text{C's } 94\tfrac{22}{37} \end{cases} = 200. \text{ Proof.}$$

---

the shares are in proportion to the products of stock and time. This may be demonstrated simply as follows, viz. Suppose 100 dollars in trade 12 months, gain 20 dollars, 50 dollars in 6 months, at the same rate, will gain 5 dollars; and both together 25 dollars. For,

$$\overline{100\times12} : \overline{50\times6} :: 20 : 5 \quad \text{and } 20+5=25.$$

Again, by composition,

$$\overline{\overline{100\times12}+\overline{50\times6}} : \overline{100\times12} :: 25 : 20 \text{ gain of \$100 in } 12m.$$

$$\text{And } \overline{\overline{100\times12}+\overline{50\times6}} : \overline{50\times6} :: 25 : 5 \text{ gain of \$50 in } 6m.$$

From which, the reason of the rule, and its correctness of principle, is evident.

EXCHANGE.

Exchange, in an arithmetical sense, is the method of reducing the money, weights and measures of one country into that of another; but particularly in fixing the momentary and actual value of money.

Gold and *silver*, as metals, have their intrinsic value; but, as they are capable of becoming the *sign* or *medium* by which merchandize may be estimated, they may receive an additional value.

Certain specific values may be given to the coins or other monies of a nation, by authority of its own government, to fix their currency amongst the subjects of that nation; but with other nations, the value of money is fixed by the current course of commerce, and the general opinion of merchants, according to the accidental circumstances of trade, money transactions between nations and the state of their public credit.

The *mutability* of the course of exchange is not *real*, but *relative;* for instance, when Philadelphia has *greater* occasion for funds in Savanna, than Savanna has in *Philadelphia*, though the price of the specie of both places bear a fixed proportional value to each other, yet when there is not a credit in Savanna equal to the *debit*, the price of *bills*, (not of money,) will rise of course at Philadelphia.

If the balance of trade be reciprocal between foreign countries, or between different sections of the same country, there will be enough of bills in the one to settle accounts with the other; in which case exchange will be at *par*. But if a nation, or one section of country, supplies another with more than it purchases, there will be a balance against that other; to discharge which, the *money* or *bills* of *exchange* on that nation, or section of country, naturally rises above *par*, and puts those against whom the balance of trade stands, below *par;* which constitutes the course of exchange. The course of exchange between two nations, publicly proclaims the state of commerce between them, and shews which of them is indebted to the other.

The nation that is in debt, as well as an individual, suffers a disadvantage both in commerce and money negociations.

The balance of trade naturally imports specie, and renders money at home more valuable abroad. On the contrary, when the balance is against a nation, its specie is exported, and becomes thereby of less value.

A bill of exchange, is a written order, delivered in one place for value received there (according to the rate of exchange agreed upon) for the like value to be paid to the holder of the bill, in the place upon which it is drawn. When the money received for a bill of exchange, is equal in value to the money paid for it, exchange is then at *par*, or equal.

The course of exchange, is the fluctuation of *par*.

Usance, is a particular time specified for the payment of a bill of exchange, after acceptance, being in some places 30 days, in others 60, 90, &c.

DOMESTIC EXCHANGE.

Domestic exchange, is a method by which the current value of money in one state, is changed to that of another.

The keeping of accounts, all over the United States, is now partly the same; they being principally kept in dollars and cents. But I shall subjoin something upon the currency of the different states, as it formerly was, viz.

The value of the Federal dollar, in the currency of the different states, stands thus:

Vermont,	6*s*.
Maine,	
New-Hampshire,	
Massachusetts,	
Rhode-Island,	
Connecticut,	
Virginia,	
Kentucky,	
Tennessee,	
New-York,	8*s*.
North-Carolina,	
South-Carolina,	4*s*. 8*d*.
Georgia,	
Mississippi ter.	
New-Jersey,	7*s*. 6*d*.
Pennsylvania,	
Maryland,	
Delaware,	

The remaining new states have principally adopted the keeping of accounts in dollars and parts of a dollar. In a very few years, I apprehend, that money reckonings will be every where kept in the same manner throughout the United States.

Vertmont,
Maine,
New-Hampshire,
Massachusetts,
Connecticut,
Virginia,
Kentucky,
Tennessee,

By adding to the current value of the dollar of these states a fourth of itself, the sum will be equal to the dollar of

New-Jersey.
Pennsylvania.
Delaware.
Maryland.

By deducting $\frac{1}{15}$ from the current value of a dollar in New-Jersey, Pennsylvania, Delaware, and Maryland, the remainder will be equal to the current value of the dollar, in each of the eight named states to the left of this page above, viz. the nominal value only.

6s.
8
4 8*d*.
7 6

These are all the nominal current values for the dollar in the respective states. And to reduce the nominal value of a number of dollars in the states where 6*s*. goes for the dollar, to the nominal value, where 8s. are equal to a dollar; $\frac{6}{8}$ reduced, becomes $\frac{3}{4}$, and $1-\frac{3}{4}=\frac{1}{4}$. Therefore to make of 6*s*. dollars, 8*s*. dollars, $\frac{1}{3}$ of their number must be added; and to make of 8*s*. dollars, those of 6*s*., $\frac{1}{4}$ of their number must be deducted.

To reduce 4*s*. 8*d*. dollars, to 6*s*. dollars, we have 4*s*. 8*d*. $=56d.$ and $6s.=72d.$; hence $\frac{56}{72}=\frac{7}{9}$, and $1-\frac{7}{9}=\frac{2}{9}$. Therefore, to make 4*s*. 8*d*. dollars out of 6*s*. dollars, $\frac{2}{9}$ parts of the latter must be deducted; if to make 6*s*. dollars out of 4*s*. 8*d*. dollars, multiply by 9 and divide by 7.

To make of 6*s*. dollars, dollars of 7*s*. 6*d*. we have $6s.=72d.$ and $7s.\ 6d.=90d.$; hence $\frac{72}{90}=\frac{4}{5}$, and $1-\frac{4}{5}=\frac{1}{5}$: therefore, to make 6*s*. dollars, 7*s*. 6*d*. dollars, multiply by 5 and divide by 4; and to make 7*s*. 6*d*. dollars, dollars of 6*s*., deduct $\frac{1}{5}$. In a similar manner, rules can be made for every possible case.

In the following page is a table of coins which formerly passed current in the United States of North America; but since November 1st, 1819, by act of congress, all gold coins of Britain, Portugal, France, and Spain, as well as other foreign gold coins, are prohibited from being a lawful tender for the payment of debts or demands. Silver coins of those countries are under the same restrictions, since April 29th, 1821.

A TABLE OF COINS.

Names of coins.	Standard weight.	Sterling money of Great Britain.	N. Hampshire, Massachusetts, Rhode-Island, Con. & Virginia.	New-York and North Carolina.	New-Jersey, Pennsylvania, Delaware and Maryland	South Carolina, and Georgia	Federal value.
(Gold.)	*dwt. gr*	£. *s. d.*	£. *s. d.*	£. *s. d.*	£. *s. d.*	£. *s. d.*	*D.cts.m.*
A Johannes,	18 0	3 12 0	4 16 0	6 8 0	6 0 0	4 0 0	16 00 0
A Half Johannes,	9 0	1 16 0	2 8 0	3 4 0	3 0 0	2 0 0	8 00 0
A Doubloon,	16 21	3 6 0	4 8 0	5 16 0	5 12 6	3 10 0	14 93 3
A Moidore,	6 18	1 7 0	1 16 0	2 8 0	2 5 0	1 8 0	6 00 0
An English Guinea,	5 6	1 1 0	1 8 0	1 17 0	1 15 0	1 1 9	4 66 7
A French Guinea,	5 5	1 1 0	1 7 6	1 16 0	1 14 6	1 1 3	4 60 0
A French Pistole,	4 6	16 6	1 2 0	1 9 0	1 8 0	18 0	3 77 3
(Silver.)							
English or French crown,	19 0	5 0	6 8	8 9	8 3	5 0	1 10 0
The Dollar of Spain, Sweden or Denmark,	17 6	4 6	6 0	8 0	7 6	4 8	1 00 0
An English Shilling,	3 18	1 0	1 4	1 9	1 8	1 0	22 0
A Pistareen,	3 11	10¼	1 2	1 7	1 6	11	20 0

All other gold coins, of equal fineness, at 89*cts.* per *dwt.*, and silver at 111*cts.* per *oz.*

A TABLE of other coins, &c. with their value in Federal money, as established some time ago, by act of congress.

	D.	*cts.*	*m*		*D.*	*cts.*	*m*
Pound sterling,	4	44	0	Guilder of the United Netherlands,		39	0
Pound of Ireland,	4	10	0	Mark Banco of Hamburg,		35	5
Pagodd of India,	1	94	0	Livre Turnois of France,		18	5
Tale of China,	1	48	0	Real Plate of Spain,		10	0
Millree of Portugal,	1	24	0				
Ruble of Russia,		66	0				
Rupee of Bengal,		55	5				

TABLE OF EXCHANGE,

For buying and selling bills of exchange above and below par.*

Rate	Amount of 1*l.* or 1$.	Rate	Amount of 1*l.* or 1$.	Rate.	Amount of 1*l.* or 1$.
¼	1.0025	4	1.04	7¾	1 0775
½	1.005	4¼	1.0425	8	1.08
¾	1.0075	4½	1.045	8¼	1.0825
1	1.01	4¾	1.0475	8½	1.085
1¼	.10125	5	1.05	8¾	1.0875
1½	1.015	5¼	1.0525	9	1.09
1¾	1.0175	5½	1.055	9¼	1.0925
2	1.02	5¾	1.0575	9½	1.095
2¼	1.0225	6	1.06	9¾	1.0975
2½	1.025	6¼	1.0625	10	1.1
2¾	1.0275	6½	1.065	10¼	1.1025
3	1.03	6¾	1.0675	10½	1.105
3¼	1.0325	7	1.07	10¾	1.1075
3½	1.035	7¼	1.0725	11	1.11
3¾	1.0375	7½	1.075	11¼	1.1125

---

* It is the custom with some, to deduct from the amount of the bill, common interest, when below par; but a plain simple example will shew the impropriety of such

The foregoing table is very convenient, for the amount of £1 or $1, at any rate per cent, from ¼ to 11¼ is at once had by inspection; therefore the amount of any bill above par is had, by multiplying the sum expressed in it, by the amount in the table, opposite to the rate per cent. If below par, divide by the tabular amount, opposite the rate per cent, and the results will be the values required.

EXAMPLES.

1. What is the amount of a bill of exchange, for $750, at 4 per cent above par?

$750 \times 1.04 = 780$ Ans.

2. What is the amount of a bill of exchange, for $780, at 4 per cent below par?

$$\frac{780}{1.04} = 750. \text{ Ans.}$$

3. What is the value of a bill of exchange, for $1175, at 9¾ per cent, above par?

Ans. $1289.56\frac{1}{4}$.

4. What is the value of a bill of exchange, for $900, at 7 per cent below par?

Ans. 841.12+

---

a practice. Suppose a bill of exchange is 5 per cent below par (its amount being $1000,) $1000 \times .05 = 50$, and $1000 - 50 = 950 =$ the value, by this way of reckoning; but $950 \times .05 + 950 = 997.50$ lacks $2.50 of 1000, (which it ought to bring,) for $\frac{1000}{1.05} = 952.38$, the true worth, and $952.38 \times 1.05 = 999.99+$ or 1000, which rule gives reciprocal advantage to both buyer and seller.

TABLE 2.

Shewing the decimal value of the parts of a £. either currency or sterling, from ¼ of a penny to 19 shillings.

¼*d.* is	£0.00104	4*s.* is	£0.2
½	.00208	5	.25
¾	.00312	6	.3
1	.00416	7	.35
2	.00833	8	.4
3	.0125	9	.45
4	.01666	10	.5
5	.0208	11	.55
6	.025	12	.6
7	.02916	13	.65
8	.03333	14	.7
9	.0375	15	.75
10	.04166	16	.8
11	.04583	17	.85
1*s.*	.05	18	.9
2	.1	19	.95
3	.15		

USE OF THE PRECEDING TABLES.

Required, the decimal value of £0 17 4½, from the above table.

Opposite to 17*s.* stands .85
to 4*d.* stands .01666
to ½*d.* stands .00208

Their sum, .86874 Ans.

NOTE. When the parts of a £ are reduced by the tables to an equivalent decimal, and annexed to the whole pounds, the sum becomes handier to work with.

EXAMPLES,

Shewing the use of the two preceding tables.

2. A, in Philadelphia, owes B, in Liverpool, £349 19 3½ sterling; to pay which, he buys a bill at 2½ per cent be-

low par; what must he pay in Federal, or United States money?

First, put down £349.95
For 3*d*. .0125
For ½*d*. .00208

Their sum will be £349.96458 Now this multiplied by 40*
40 and divided by 9, gives the true value Federal.

9)13998.5832

$1555.39813 Ans.=1555.40 nearly.

3. What is the value of a bill amounting to $1300, at 2½ per cent below par?

Amount of bill 1300.
Number in table 1. opposite 2½=1.025 and their quotient
=1268.39+ Ans.

For 1268.39×1.025=1299.999 or 1300. Proof.

4. A, in Philadelphia, owes B, in London £1000 sterling, to pay which, he buys a bill at 4 per cent below par; how many dollars will he have to pay for the same? Ans. $4273.50.

5. A factor, in Philadelphia, owes a merchant in Liverpool, £1500 sterling; to pay which, he buys a bill, at 4 per cent above par; how many dollars did the bill cost him? Ans. $6933⅓.

Let these examples be proved by reducing their prices to sterling money.

A merchant may turn both rise and fall of exchange to his advantage, by buying and remitting when it is low; establishing a fund for which, he may draw to advantage when exchange is high.

EXAMPLE.

A merchant, in Philadelphia, remits a bill of $5000 to London, which he purchased at 2½ per cent below par; but on the rise of exchange, he drew for the same at 2½ per cent above par. What did he gain? Ans. 539.63\frac{29}{41}$.

---

* £1 sterling is equal to 33*s*. 4*d*.=400 pence, and one dollar is equal to 90; therefore, $\frac{400}{90}=\frac{40}{9}$, the ratio that the pence in the sterling £, bears to the pence in a $.

Great Britain exchanges with the following places, viz. Switzerland, Nuremburg, Leipsic, Dresden, Osnaburg, Brunswick, Cologn, Liege, Strasburg, Cracow, Denmark, Norway, Riga, Revel, and Narva, on the rix dollar; the par being 4*s.* 6*d.*, or 54*d.* sterling.

With Ireland, at 8⅓ per cent advance.		
France, for the ecu of exchange	at 29.365*d.* sterling.	
Holland, Flanders, Brabant, for 36 schil 7 groot	at £1.	
Hamburg, Antwerp, for 35 sch 6¾ groot	at £1.	
Spain, for the piaster of exchange or 8 rials	at 43*d.*	do.
Portugal, on the milrea	at 67½	do.
Venice, on the ducat of 20 sols d'or	at 50½	do.
Genoa and Leghorn, on the Pezzo	at 54	do.
Rome, one the crown	at 73½	do.
Naples, on the ducat	at 40½	do.
Florence, on the crown	at 64½	do.
Bologna, on the dollar	at 51	do.
Sicily, on the crown	at 60	do
Vienna, on the rix dollar	at 56	do.
Bremen, on the rix dollar	at 42	do.
Breslaw, on the rix dollar	at 39	do.
Berlin, on the rix dollar	at 48	do.
Frankfort, on the main, on the florin	at 36	do.
Stetin, on the mark	at 18	do.
Embden, on the rix dollar	at 42	do.
Bolsena, on the rix dollar	at 44	do.
Dantzick, on 13½ florins	at £1	do.
Stockholm, Upsal, Thorn, on the rix dollar	at 54*d.*	do.
Petersburg, on the ruble	at 53	do.

More or less, according to the course of exchange.

EXCHANGE WITH IRELAND.

The par between England and Ireland is 8⅓ per cent; but the course of exchange is from 5 to 12 per cent.

Bills on Ireland are usually drawn for sterling money, (accounts between England and Ireland being kept in the same denominations.)

To change Irish currency into English at par,

RULE.

From the Irish subtract $\frac{1}{13}$ part, the remainder will be English.

EXAMPLE.

£108 6 8 Irish, being equal to £100 English; the Irish £108 6 8 contains 13 parts, each equal to £8 6 8, and the £100 English contains just 12 such parts. For, 8 6 8×13=108 6 8, and 8 6 8×12=100; therefore it is plain, that if $\frac{1}{13}$ be subtracted from Irish, it becomes English, or sterling; and for the same reason, if to English, or sterling, $\frac{1}{12}$ be added, it will be equal to Irish. Therefore, to change English into Irish currency at par,

RULE.

To the English, add a $\frac{1}{12}$ part, and the sum will be Irish.

One dollar, Irish currency, being 4*s.* 10½*d.* equal to 117 half-pence, it is plain, that if the half-pence of any sum Irish, be divided by 117, the quotient will be equal to the dollar which it is worth: and if any number of dollars be multiplied by 117, the product will be equal to the Irish half-pence in such dollars.

EXAMPLES.

1. How many dollars are in £100 Irish?

```
£100
  20
 ----
2000
  12
 -----
24000
    2
 -----
117)48000(410.25 75/117 Ans.
    468
    ---
     120
     117
     ---
       300
       234
       ---
        660
        585
        ---
         75
```

And $410.25\frac{75}{117}$×117=48000 Irish half-pence=£100.

Or, to multiply Irish by 160 and divide by 39, we get the same result; for 4*s.* 10½*d.* is equal to 117 half-pence, and £1 is equal to 480 half-pence. Therefore, $\frac{480}{117}=\frac{160}{39}$=ratio.

2. How many dollars are equal to £1000 Irish, exchange at par? Ans.

3. In $10000, How much Irish currency? Ans.

EXCHANGE WITH FRANCE.

MEASURES AND WEIGHTS.

The scale of measures, in France, is established on the measure of a degree of the meridian, equally distant from the north pole and the equator, which is found to be 57027 toises, or fathoms; this multiplied by 90°, gives 5132430 toises, or 30794580 feet, from the pole to the equator, and is the assumed unity expressed in ancient measure, from the iron toise of the French philosophical academy, and the origin from which all their measures are derived.

This being established, they divided 30794580 by 10000000, which produced 3.0794580, or 3 feet 11½ inches nearly; and this portable measure was adopted to take place of the foot, the toise, and the ell, and called a *metre.*

Proceeding from this measure, as the common unity, in a decuple or ten-fold ratio, they assumed the following names, viz.

	French feet.	*ft.*	*in.*	*lines.*
Metre,	3.079458=	3	0	11½
Decametre,	30.79458=	30	9	7
Hectometre,	307.9458=	307	11	10
Kilometre,	3079.458=	3079	10	4
Myriametre,	30794.58=	30794	7	4

Subdivisions of the metre, viz.		*in.*	*lines.*	*points.*
Decimetre,	.3079458=	3	8	4
Centimetre,	.03079458=	0	4	5
Millimetre,	.003079458=	0	0	5⅓

From these last measures are deduced

ITINERARY, OR LONG MEASURES.

	1 Decametre,	=	5. toises.
10 Decametre,	1 Hectometre,		51.
10 Hectometre,	1 Kilometre,		513.
10 Kilometre,	1 Myriametre,		5132.

AGRARIAN, OR LAND MEASURE.

1 square decametre, or 100 square metres, make 1 are.
10000 sqaure metres, 100 square ares, 1 hectare.

These are substituted for the perch and acre (arpent) of France.

CUBIC MEASURES.

1 cubic metre is called a stere; the 10th part of a stere is called a decistere, and is used in measuring carpentry, &c. A vessel of a cubical form, whose side is one decimetre, or a cylindrieal vessel of the same capacity, or solid content, is called a libre; it contains about *2lb.* of water, or 25 ounces of wheat, and is assumed as the element, or standard, for measures of capacity.

10 decilitre make 1 litre.
10 litre, 1 decalitre.
10 decalitre, 1 hectolitre.
10 hectolitre are equal in capacity to 1 cubic metre

WEIGHT.

The original weight is taken to be equal to the quantity of disstilled water contained in a cubic vessel, the side of which is equal to the one hundredth part of a metre. This water weighed in vacuo, at the temperature of melting ice, is equal to 18.841 grains; to this weight is given the denomination of gramme; and from the gramme, all the superior and inferior weights are deduced, by multiplication and division.

	milligramme,	.001
10 milligramme,	1 centigramme,	.01
10 centigramme,	1 decigramme,	.1
10 decigramme,	1 gramme,	1.
10 gramme,	1 decagramme,	10.
10 decagramme,	1 hectogramme,	100.
10 hectogramme,	1 kilogramme,	1000.
10 Kilogramme,	1 myriagramme,	10000.

We see, therefore, that the gramme is an unit placed between two series of like numbers; the one increasing, in a decuple ratio, to the myriagramme, by which the weight of goods and wares may be ascertained; the other decreasing in the same ratio, to the milligramme, or $\frac{1}{1000}$th part of gramme; and is, therefore, adapted to medicinal and monotary weights and mixtures.

THE PRECEDING COMPARED WITH ENGLISH MEASURES.

The metre is equal in length to	39.383 inches.
A litre,	61.083 cubic inches.
A gramme,	22.966 grains Troy.
A kilogramme,	3*lb*.4½*oz*. avoirdupoise
A hectare,	11968 square yards, not quite 2½ acres.

EXAMPLES.

1. What is the content of 750 litres, in wine gallons?

$$\frac{61.083 \times 750}{231} =$$

2. In 120 metres, how many yards?

$$\frac{39.383 \times 120}{36} =$$

The French money of account, is francs and centimes. The real monies are connected with the general system of weights, &c. and are as follow:

FRENCH COPPER COINS.

1 centime,	weight	1	gramme.
5 centimes,		5	do.
1 decime,		10	do.
2 decimes,		20	do.

FRENCH SILVER COINS.

1 franc,	weight,	5	grammes.
5 francs,		25	grammes.

Gold is coined into pieces of 10 *grammes.*

A franc, equal to the ancient livre turnoise, = 19*cts*.
A 5 franc piece, like a crown, = 95*cts*.

To change francs into dollars,

RULE.

Multiply the francs and centimes, by the rate of exchange;* divide by 100, and the quotient will be dollars.

EXAMPLES.

1. What is the value of 5000 francs, 40 centimes, at 18 cents per franc? Ans.

2. A merchant in Bourdeaux, owes a merchant in Philadelphia 9000 francs, 33 centimes; what is the value in federal dollars, at 19*cts.* per franc? Ans.

To reduce federal dollars to francs,

RULE.

Reduce the dollars to cents, and divide by the rate for a franc; if the quotient consists of a whole number and a fraction, continue the division two places, for centimes.

EXAMPLES.

1. A merchant in Bourdeaux, owes a merchant in Philadelphia, nett proceeds of a consignment amounting to 750 dollars, 16 cents; how many francs must he draw for, if exchange is at 19 cents per franc?

$$\frac{750.16 \times 10}{19} =$$

2. A, in Philadelphia, draws on B, of Paris, for 5000 francs, at 18 cents per franc. Ans. $900.

3. If A, in Philadelphia, buys a draft on Brest, of 4444 francs, 44$\frac{4}{9}$ centimes, for 800 dollars, what was the course of exchange? Ans. 18*cts.* per franc.

FORM OF A BILL OF EXCHANGE.

Philadelphia, June 16, 1820.

Exchange for 8059*fr.* 60*ct.*

At usance pay this first of exchange (second of the same tenor and date unpaid) to Brusieur and Lamotte, or order, eight thousand and fifty-nine francs, sixty centimes, value received, and charge the same to account, with or without advice.

---

* The number of federal cents which are equal to a franc.

JAMES SIMMONS.

Jean Bornorilli,
Philadelphia.

What is the value of the above bill, at 20 cents per franc?

EXCHANGE WITH SPAIN.

The monies of Spain are of two sorts, viz. the one called *velon*, the other, plate money. The velon is to the plate money, as 34 to 64. Dealers and commissioners of excise, keep their accounts in rials and *mervadies velon*, viz.

2 mervadies velon	make	1 quartile.
4 do.	do.	1 quarto.
8½ quartos, 34 mer. vel.		1 rial velon.
15 rials velon, or 510 do.		1 peso or current dollar
20 do. or 680 do.		1 hard dollar.
16 quartos or 64 do.		1 rial plate.

But some bankers, remitters, and some merchants, keep their accounts in old plate, or money of exchange, viz.

34 mervadies plate make	1 real plate, value in U.S.	$0.10
8 rials plate	1 piaster of exchange,	.80
10 do.	1 dollar,	1.
11 do.	1 ducat of exchange,	1.10
32 do.	1 doubloon of exchange,	3.20

In the following table, the first column shews the assay to be worse than American or British gold and silver. *Second*, the absolute weight. *Third*, the standard weight. *Fourth*; its value in federal money.

Silver.	*dwt.*	*dwt.*	*gr.*	*dwt.*	*gr.*	*value.*
Peso, or piaster of 8 rials, now reduced to 10,	1	17	12	17	10.2	$1
Pillar, piece of 8 rials, now reduced to 10,	standard.	17	9	17	9	1 nearly

Silver.	*dwt.*	*dwt.*	*gr.*	*dwt.*	*gr.*	*value.*
Mexico piece,	1	17	10	17	8.14	99⅜*cts.* current at 1 dollar.
New Seville piece of 8,	1½	14	00	13	21.15	79⅘*cts.* current at 80 cents.
Pistrine of 2 rials plate,						20 cents.
Gold.						
Old Spanish doubloon,	½	17	8	17	5.12	These coins were formerly current in the U.S. at 87½*cts.* the *dwt.* nearly.
half do.	½	8	16	8	14.16	
Pistole,	½	4	8	4	7.8	
New Seville double pistole	3	8	16¾			
Pistole,	3	4	8⅜			

To reduce rials velon to rials of exchange.

RULE.

Multiply the rials velon by 8½, the quartos in a rial velon, adding 1 for every 4 mervadies, divide by 16; the quotient will be the rials exchange.

EXAMPLES.

1. Reduce 5740 rials 24 *mer. vn.* to rials of exchange or plate.

$$\begin{array}{l} 5740 \\ \quad 8\frac{1}{2} \text{ quartos in a rial velon.} \\ \hline 16)48796 \text{ product, including 9 for the 24 } \textit{mer. vn.} \\ \hline \quad 3049\frac{3}{4} \text{ rials of exchange. Ans.} \end{array}$$

2. In 17469 rials 18 *mer. vn.* how many dollars plate? Ans.

3. In 1000000 rials velon, how many rials of exchange? Ans.

To reduce rials velon to rials of exchange,

RULE.

Multiply the rials of exchange by 16, the quartos in a rial of exchange; add 1 for every two mervadies, divide this product by 8½, the quotient will be the rials velon.

1. Reduce 6450 rials 20 mervadies plate to rials velon.
8½)6450 × 16(=17)103210 × 2(12242*ri.* 12*mer.*
Or, 8.5)103210(Ans.
2. Reduce 2425 rials exchange to rials velon. Ans.
3. In 1000 rials of plate, how many rials velon? Ans.

To reduce Spanish to federal,

RULE.

Reduce the odd rials and mervadies to the decimal of a piaster, multiply the given Spanish by .80; the amount will be dollars, and the decimal of a dollar, of which two places will be sufficient to retain for cents.

EXAMPLE.

A, in Valentia, is indebted to B, in Philadelphia, 875 piasters 6 rials 25½ mervadies plate; what number of dollars is A charged in B's books?
875.6.25½=875.84375&875.84375 × ·80=700.67½

TABLE

OF MERVADIES AND RIALS EXPRESSED IN THE DECIMAL OF A PIASTER.

mer.		mer.		mer.	
1	.00367	15	.05514	29	.1066
2	.00735	16	.05881	30	.11028
3	.01102	17	.06249	31	.11395
4	.0147	18	.06616	32	.11763
5	.01838	19	.06984	33	.12131
6	.02205	20	.07352		
7	.02573	21	.07719	Rials.	
8	.0294	22	.08087	1	.125
9	.03308	23	.08455	2	.25
10	.03676	24	.08822	3	.375
11	04043	25	.0919	4	.5
12	.04411	26	09557	5	.625
13	.04778	27	.09925	6	.75
14	.05146	28	.10293	7	.875

USE OF THE PRECEDING TABLE.

Find the value of 6 rials, 17 mervadies, in the decimal of a piaster.

6 rials	.75
17 mervadies	.06249
	.81249 Ans.

To reduce Federal to Spanish,

RULE.

Divide the Federal by .80; the quotient will be Spanish piasters, and the decimal of a piaster; the decimal should continue to 4 or 5 places.

EXAMPLES.

Philadelphia, June 20, 1820.

Exchange for $6403.75.

At twenty days sight, pay this second of exchange (first and second of the same tenor and date unpaid) to Samuel Cunningham or order, six thousand four hundred and three dollars, seventy-five cents, value received, which charge to account as per advice.

Don Jean Cordillo,
merch't Barcelona.
DAVID JONES.

Required the amount in Spanish piasters.

A merchant in Philadelphia draws on Madrid for 1700 dollars; how many piasters will be paid for the same?

Ans.

Sold a bill of exchange for 5000 piasters 4 rials 12 mervadies plate; what is the amount, at 75 cents per piaster?

Ans.

Barcelona, June 19, 1820.

Exchange for 640*p.* 4*r.* 24*merv.*

At sixty days sight, pay this third of exchange (the third and fourth of the same tenor and date unpaid) to Don Alonzo Viola, or order, six hundred and forty piasters, four rials, twenty-four mervadies plate, and charge the account as per advice. CHARLES DE ALVAREY.

Messrs. Beck & Lamot,
Philadelphia,

How much Federal money should be paid for the preceding bill? Ans. $512.47.

Sold a draft of Don Jean de Mona, at 5 per cent above par, for which I received 1223 dollars 51 cents; how much should he charge me in piasters?

Ans. 1456*p*. 4*r*. 16*mer*.

The following example will serve to shew how such cases may be resolved, without using decimals.

In 6478 dollars 33 cents, how many piasters?

.80)6478.33(8697 Piasters.

10)73 remains.

—(7 rials.

Remainder 3

× 3⅖

10⅕ mervadies.

To change the above to Federal,

RULE.

Multiply the piasters by .80, add 10 cents for each rial, and 2½ for every 8½ mervadies; and divide the sum by 100.

8097*p*. 7*r*. 10⅕*mer*.

.80

647760

70 = 7 rials.

2½ = 8½ mervadies.

½ = 1½ do.

6478.33 Ans.

EXCHANGE WITH PORTUGAL.

The Portuguese keep their accounts in milreas and reas; 1000 make 1 milrea, valued in the United States at 1 dollar 25 cents.

The milrea is an imaginary piece of account; the real monies of Portugal are as follow:

Silver.

Crusado of 400 reas, not stamped, value in the U.S.		$0.50
Crusado of 480 reas, stamped in 1643,		60
12 vintin piece, or half crusado of 240 reas,		30
5 vintin piece,	of 100 do.	12½
2½ vintin piece,	of 50 do.	6¼

Note. In estimating invoices for the calculation of American duties, it is ordered, that the milrea shall be valued at 1 dollar 24 cents; but from the intrinsic value of the Joannes, the milrea seems to be justly valued at 67½*d.* sterling, or 125 cents, Federal money, exactly.

Gold.	m.	r.	*D.ct.*
Double Joannes, or piece of	25*m*.	600*r*. value in the U.S.	32.
Single do.	12	800	16.
Half do.	6	400	8.
Quarter do.	3	200	4.
Eighth do.	1	600	2.
Testoon, or $\frac{1}{16}$ do.	0	800	1.
Moidore,	4	800	6.
Half moidore,	2	400	3.
Quarter do.	1	200	1.50

To change reas and milreas into dollars and cents,

RULE.*

To the milreas and reas add one-fourth part, the sum will be dollars and cents.

EXAMPLE.

In 5000*m*. 400*r*. how many dollars?

Operation. $5000.400+\frac{5000.400}{4}=\$6250.50$ Ans.

---

* Because a milrea is equal to a fourth of itself more than a dollar, the parts of a milrea have the same ratio to the parts of a dollar, that their wholes have; therefore, any number of milreas and rials, increased by one-fourth of themselves, will be equal the dollars and cents, in such reas and milreas.

To change dollars and cents into milreas and reas.

RULE.

Subtract $\frac{1}{5}$ of the dollars and cents from themselves, and the remainder will be milreas and reas, (a milrea being $\frac{5}{4}$ of a dollar.)

EXAMPLE.

Reduce 6250 dollars 50 cents to milreas and reas.

Operation. $\frac{1}{5}$|6250.500
1250.100 deduct.

5000.400 Ans.

Note. In changing dollars into milreas, let a cypher be annexed to the cents, if of 2 places, if but 1, let two be annexed, that the reas may have three places.

What must be paid for a draft of 900 milreas 280 reas, at 1 dollar 24 cents per milrea? Ans. $1116.34.72+

Philadelphia, June 20, 1820.

Exchange for 6000*m*. 400*r*.

Sixty days after sight, pay this first of exchange (second and third of the same tenor and date unpaid) to Joannes de Silva, or order, six thousand milreas, four hundred reas, value received, and charge the same to account, as per advice. ALFRED PRINCE.

Messrs. Monteiro Del Callo &
company, merch'ts, Lisbon.

Required, the amount in dollars, at 5 per cent above par. Ans. $7875.52½.

Sold my draft on Alphonso de Consillio, at 3 per cent under par, for which I received 6552 dollars; how many milreas and reas was the bill drawn for?

Ans. 5398*m*. 848*r*.

EXCHANGE WITH HOLLAND,

THE BATAVIAN REPUBLIC, OR SEVEN UNITED PROVINCES.

There are two banks in Holland; one in Amsterdam, the other in Rotterdam. That in Amsterdam, (the most considerable in Europe) was established on the 31st of

January, 1609, by the authority of the states general, and under the direction of the Burgomasters of the city, who having constituted themselves perpetual cashiers of the merchants of Amsterdam, are the security for the bank.

By the establishment of this bank, it is ordained, that bills of exchange and wholesale goods, shall all be paid in bank, unless the sum be under 500 guilders; in which case, it cannot be entered in the bank under a duty of 6 stivers, except by the East or West India company.

No director can use the freedom of extracting money from the bank, even for a single day, on pain of death. The strictness and fidelity with which this bank is kept, is productive of so much security and despatch, that a bank payment is reckoned from 3 to 5 per cent better than cash, beside a premium allowed the bank for every deposit.

The bank of Amsterdam is supposed to contain the greatest treasure in the world, of gold and jewels. The specie kept here, is computed at 36 millions sterling.

In Holland, they keep their accounts in guilders, stivers, and pennings.

2 groot, or 16 pennings, make	1 stiver,
20 stivers,	1 guilder, or florin.
6 guilders;	1£. or pound Flemish.

Or in pounds, shillings, and groots.

8 pennings make	1 groots.
12 groots	1 shilling.
20 shillings	1£. Flemish.

The par of a Dutch guilder, florin, or piece of 20 stivers, is calculated by sir Thomas Bond (a late writer) at 21.86*d.* sterling; which, in Federal money, is 40½ cents, nearly. This seems something above its intrinsic value; for the true par may be easily estimated, from sir Isaac Newton's table of the assay weight and value of foreign coin, published in London, by order of the Privy Council, 1740, viz.

Standard weight, 6*dwt.* 17.05*gr.*; assay, 2*dwt.*, worse than British or American silver. Valued in London, at 20.08 pence, sterling, and in the United States, about 37 cents.

The United States exchange with Holland, at 40 cents the guilder, more or less; but from the above comparison, it appears reasonable, the par should be fixed at 37 cents.

RULE.

Multiply the guilders by the rate of exchange, and divide by 100; the quotient is the answer in dollars. If stivers and pennings occur, take practical parts, or otherwise reduce them to the decimal of a guilder.

EXAMPLES.

1. A merchant in Philadelphia sold his bill on Amsterdam for 1000 guilders, 15 stivers, 8 pennings, at 40 cents per guilder; what was the amount in Federal money?

1000.
15*st.* 8*pen.*= .775
And 1000775 × .40 = $400.31 Ans.

2d method. 1000.15.8
40

40000
$10 = \frac{1}{2}$ 20
$5 = \frac{1}{2}$ 10
8*pen.* $= \frac{1}{10}$ 1

$400.31 Ans.

2. A, in Amsterdam, is indebted to B, in Philadelphia, 3400 guilders; what is the amount at 35 cents per guilder? Ans. $1190.

3. C, in Philadelphia, owes D, in Amsterdam, 2000 dollars; how many guilders will it amount to, at $33\frac{1}{3}$ cents the guilder? Because $33\frac{1}{3}$ is $=\frac{1}{3}$ of 100,
$2000 \div \frac{1}{3} = 2000 \times \frac{3}{1} = 6000$*guil.* Ans.

4. In 1500 guilders, how many dollars, at $33\frac{1}{3}$ cents per guilder? Ans. $500.

Note. A general rule to be observed in remitting to Holland is, to make them when guilders bear the lowest price there, and fix a fund which may be drawn upon to advantage when they rise.

Holland exchanges with France upon the French crown, for 54 groots, more or less; Hamburg, upon the dollars, for 32 Flemish shillings; Spain, upon the ducat, for 375 mervadies, for 97 groots; with Portugal upon the crusado of 400 reas, for 44 groots; Genoa, upon the piaster of 5 livers banco, for 92 groots; Venice, upon the duçat of 24 gross banco, for 88 groots; Leghorn, upon the piaster of 20 sols d'or, for 86 groots; England, upon the £. sterling, for 36*s*. 6*d*. Flemish, more or less, according to the rate of exchange.

EXCHANGE WITH AUSTRIAN NETHERLANDS.

Antwerp, or Anvers, is the principal place of exchange in Austrian Netherlands, and was once the metropolis of the seventeen provinces; but its commerce was ruined by the Dutch, who closed the navigation of the Scheldt, soon after they threw off the Spanish yoke. This was the more cruel, as the people of Antwerp had been their allies and fellow sufferers in the cause of liberty. The change of politics in France effected a happy change for the people of Antwerp; for their commercial privileges are again restored, and Antwerp bids fair to rival the most flourishing commercial cities on the continent.

Antwerp is pleasantly situated on the Scheldt, which is 20 feet deep, at low water, and rising 20 feet more at flood; allows vessels of the greatest burthen to come up to the quays and unload.

Their money calculations and exchange, are the same as in Holland; therefore, no other examples are necessary.

EXCHANGE WITH HAMBURG.

Hamburg is one of the most flourishing cities in Europe; it has the sovereignty of a small district, about 10 miles in circuit; and though the king of Denmark affects certain privileges within its walls, yet it may be considered as a well regulated, independent commonwealth. It is well situated for trade, on the Elbe, which forms two spacious harbours. The stately public and private edifices, the number of canals and bridges, give the town an air of grandeur, not to be surpassed by many towns in Europe.

The funds of the bank are inferior to those of Amsterdam; yet the integrity with which it has been conducted, has gained a very high reputation.

The inspection of this bank comes not under the inspection of the senate; but as the corporation and citizens are their sureties, they choose their directors by a majority of votes. Though the capital of this bank be very considerable, it is uncertain the amount; as the clerks and cashiers are bound to secrecy by oath; therefore, no seizure can be made on any deposit, or bank account, as no one knows how another stands with the bank.

They keep their accounts in rix dollars, sous, and denier lubs.

12 denier lubs make	1 sous lub, value in the U.S.	$0.0208125
16 sous lubs,	1 mark banco (or mak lub)	.33⅓
3 marks banco,	1 rix dollar,	1.
2½ rix dollars,	1 £. gross,	2.50

Or, rix dollars schillings, and denier gross.

6 denier lubs make	1 denier gross,	.0104
2 denier gross,	1 sous lub,	.0208
6 sous lub,	1 schilling,	.125
8 schillings,	1 rix dollar,	1.
2½ rix dollars,	1 £. gross,	2.50

The money of exchange, generally called bank or banco, on account of its convenience and undoubted security in trade, is esteemed better than the current, from 3 to 6 per cent, which excess is called agio. The current monies are said to have been so much adulterated of late, that the agio has risen to 15 per cent, and upwards; but all bills of exchange are paid in bank.

To change banco into currency,

RULE.

100 banco : 100 + agio : : so is given banco : currency.

The United States exchange with Hamburg at 33⅓ cents per mark banco, three marks being equal to one dollar; but the course of exchange is from 27 to 35 cents per mark banco.

To change currency to banco,

RULE.

100 + agio : 100 : : given currency : banco.

To reduce Federal to marks banco,

RULE.

Reduce the Federal to cents, and divide by the rate of exchange, for the marks banco.

To reduce marks banco to Federal,

RULE.

Reduce the sous and deniers to the decimal of a mark, and then multiply by the rate of exchange for the cents; or, multiply by the rate of exchange, and take parts for the lower denominations.

EXAMPLES.

1. A bill of exchange on Hamburg, for 12546 marks, 10 sous, 8 deniers banco, is sold at 30 cents per mark; what is the amount in dollars?

12546 — 12546.6′ this 6′ = $\frac{2}{3}$

8 sous = $\frac{1}{2}$ 30 — 30

376380 — $3764.000 Ans.

2 = $\frac{1}{4}$ 15

6d. [illegible] $\frac{1}{4}$ 3$\frac{3}{4}$ =

2 = $\frac{1}{3}$ $\frac{15}{16}$

$\frac{5}{16}$

$3764. Ans.

2. A, in Philadelphia, draws on Hamburg, for 11564 marks banco, 8 sous, 10 deniers, and receives at the rate of 33$\frac{1}{3}$ cents per mark banco; what is the value in dollars?

Ans. $3854.85

3. C, in Baltimore, draws on D in Hamburg, for 12546 marks banco, 10 sous, 8 deniers, in favor of E, who pays him 3764 dollars; what was the course of exchange?

Ans. 30*cts*.

Hamburg exchanges with
France, upon the crown of 60 sols, for 29 schilling lubs.
Spain, upon the crown of 375 merv., for 93 gross.
Portugal, upon the crusado of 400 reas, for 42 gr.
Venice, upon the ducat of 24 gross, for 86 gross.
England, upon the £ sterling, for 35.6⅓s. Flemish.

More or less, according to the course of exch.

EXCHANGE WITH DENMARK, NORWAY, &c.

The exchanges with Denmark and Norway, are intrinsically the same with that of Hamburg. Their monies being of the value, divided in the same manner, and nearly of the same denominations, viz.

12 pennings make	1 sch. lub, value in the U.S.	$0.0208125
16 schillings,	1 mark,	.33⅓
3 marks,	1 rix dollar,	1.
6¼ marks,	1 ducat,	2.08⅓

The examples and questions in exchange with Hamburg, are sufficient for this.

For exchange with Sweden and Russia, see the general tables.

EXCHANGE WITH GENOA, LEGHORN, FLORENCE, AND CORSICA.

Saint George's bank, in Genoa, is constituted of such parts of the public revenue, as have been mortgaged by government, during the exigencies of the commonwealth. It is become a kind of inferior senate, of powerful sway in the republic, and often breaks the uniformity of their aristocratic government. The president's office is during life.

Accounts are kept in the bank, in pezzoes or piasters, soldi, and denari.

12 denari make	1 soldi, value in the U. S.	$0.05
20 soldi,	1 pezzo, piaster, or dollar,	1.

This is the money of exchange; but, in general accounts are kept in lire, soldi, denari, divided as before. The money of Leghorn, is equal in value only to $\frac{1}{5}$ of the exchange; and the livre of Genoa, is only $\frac{1}{5}$ value of the exchange money. The par of a piaster, lire money, is only 20 cents.

12 denari	make 1 soldi,	value in the U.S.	$0.01
20 soldi,	1 lire,		.20
4 do.	1 chevalet,		.04
30 do.	1 testoon,		.30
6 testoons,	1 geroni,		1.80

To reduce lire money to money of exchange,

RULE.

Divide the lire money by 5, the quotient will be the money of exchange.

EXAMPLE.

In 794 pezzoes, 19 soldi, 6 denari lire, how much money of exchange?

$$\begin{array}{r} 5)794\ \ 19\ \ 9 \\ \hline 158\ \ 19\ \ 10\frac{4}{5} \text{ exchange.} \end{array}$$

To reduce money of exchange to lire money,

RULE.

Multiply the money of exchange by 5, the product will the lire money.

EXAMPLE.

In 684 pezzoes, 15 soldi, 9 denari, exchange, how many lire money?

$$\begin{array}{r} 684\ \ 15\ \ 9 \\ 5 \\ \hline 3423\ \ 18\ \ 9 \text{ lire.} \end{array}$$

To reduce pezzoes, soldi, and denari to Federal money,

RULE.

Reduce the soldi and denari to the decimal of a pezzo, and the exchange money will answer to dollars and cents at par.

EXAMPLE.

In 648 pezzoes, 7 soldi, 6 denari, how much Federal money?

6 denari = .005
7 soldi = .37
pezzoes 648.

$648.375 Ans.

NOTE. This currency, or money, being divided like pounds, shillings and pence, to find the decimal value of the lower denominations, use table 2.

Otherwise—Take the pezzoes as so many dollars, and for the soldi and denari, take parts of a dollar, for the cents.

Foregoing example. Pezzoes 647.
5sol. = ¼ .25
2s. 6den. = ½ .12½

$648.37½ Ans.

To reduce dollars and cents to pezzoes, soldi and denari,

RULE.

Take the dollars as so many pezzoes; multiply the cents by the lower known names in the exchange money, for soldi and denari, &c.

EXAMPLE.

Reduce 847 dollars, 37½ cents, to pezzoes, &c.

847.375
20

7.500
12

6.000

847*pez.* 7*sol.* 6*de.* Ans.

When the exchange is so many cents per pezzo above or below par,

RULE.

Reduce the lower denominations to the decimal of a pezzo, and multiply by the rate of exchange.

EXAMPLE.

What is the value of 500 pezzoes, 14 soldi, 6 denari, in Federal money, at 90 cents per pezzo? Ans. $450.65¼.

Exchange for 4000*pez.* 10*sol.* 6*den.*

At usance, pay this first of exchange (second of same tenor and date unpaid) to Pier de Foscari, or order, four thousand pezzoes, ten soldi, six denari, exchange, which charge to account as per advice. G. L'ESTRANGE.

To Messrs Francisco, & co.
merchants, Leghorn.

What is the value of the above, at $1.04 per pezzo?
Ans. $4160.54⅗.

EXCHANGE WITH VENICE.

The bank of Venice, or banco del Giro, is a public depository of the merchants' money, as established by an edict of the republic, ordering the sale of merchandize by wholesale, and bills of exchange, to be made in bank. All debtors and creditors are obliged to pay or receive their money or accounts in bank by transfer, from one account to the other.

A ready money account has been opened for the accommodation of strange merchants, and to supply the necessities of a real trade. This, instead of diminishing the funds, has been found to augment them. The funds of the bank were fixed at 5000000 ducats.

Their monies are as follow:

5¼ soldi	make	1 gross,	value in the U.S.
24 gross,		1 ducat,	

But merchants and bankers, for the sake of ease in calculation, keep their accounts in ducats, sols, and deniers d'or.

12 deniers d'or	make	1 sol,	value in the U.S. $0.0465+
20 sols,		1 ducat,	.9305

But 93 cents are near enough for calculation.

This money of exchange is imaginary; 100 ducats whereof, make 120 ducats current; the difference is called agio.

The money of *Venice* is of three sorts, viz. Bank, banco-current, and picoli. The bank money is 20 per cent better than the *picoli;* but all bills of exchange are paid in bank.

	Assay.	Absolute *wt.*	
Silver.	*dwt.*	*dwt.*	*gr.*
The old ducat of Venice	23½	14	15
New do. with No. 124, or 124 sols,		18	2
Crusado, St. Mark, with No. 140 or 140 do.		20	6
Piece of 20 jewels,	b. 6	3	177+

Gold.		*dwt.*	*gr.*	*m.*
Double ducat,	b. 1½	4	18	8
Single do.		2	9	4

To Reduce Venitian ducats, soldi, and denari, to Federal money,

RULE.

Reduce the soldi and denari to the decimal of a ducat; then multiply the given sum by .93, the product will be dollars and cents.

NOTE. This money being divided as currency, the decimal parts for the lower denominations, will be found in table 2.

EXAMPLE.

In 3254 ducats, 16 sols, 6 deniers, how many dollars, if 93 cents be a par for one ducat?

6 denari = .025
16 soldi = .8
ducats, 3254.

3254.825
93

9764475
29293425

3026.98.725 Ans.

To reduce Federal money to Venitian ducats, &c.

RULE.

Divide the dollars and cents by .93, the quotient will be ducats and decimal parts of ducats; then find the value of the decimal in the known denominations.

EXAMPLE.

Bought a bill of exchange on Venice, amounting to $3464.50; what is the amount in Venitian ducats of exchange, at 93 cents per ducat? Ans. *3725du. 5s. 4½de.*

```
93)3464.50(3725.2688
                  20
                ----
              5.3760
                  12
                ----
              4.5120
           ---------
  Ans. 3725.5.4.512
```

Philadelphia, July 16, 1820.

Exchange for *750duc. 16sol. 6den. d'or.*

At usance, pay this second of exchange (first, of the same tenor and date unpaid) to Messrs. Borelli & co. seven hundred and fifty ducats, sixteen sols, six deniers bank; which charge to account with or without advice.

JONATHAN BURRALL.

To Messrs. Duvilla Don, & co. }
merchants, Venice. }

Required, the amount of this draft, at 90 cents per ducat. Ans. $675.74¼.

Having proceeded minutely with the most particular commercial nations, and (I conceive) sufficiently illustrated the different monies and exchanges, I deem it useless to pursue calculation farther. But as Exchange is a subject so extensive, its consideration so useful and advantageous to the young, as well as to the old merchant, I shall here subjoin tables of the different monies in use with the several trading nations of the world; the foregoing rules and applications being sufficient directions for the different exchanges that may be required.

TABLES OF EXCHANGE.

PIEDMONT, SAVOY, AND SARDINIA. TURIN, CHAMBERRY, AND CAGLIARI.

12 denari make	1 Soldi, value in the U.S.	$0.0138+
12 soldi,	1 florin,	.166
20 soldi,	1 lire,	.277
6 florins,	1 scudi,	1.
7 florins,	1 ducatoon,	1.166
13 lires,	1 pistole,	3.611
16 lires,	1 louis d'òr,	4.444

MILAN, PAVIA, MODENA, PARMA.

12 denari make	1 soldi,	$0.
20 soldi,	1 lire,	.162
115 soldi,	1 scudi cur.	.931
117 soldi,	1 scudi ex.	.944
6 lires,	1 philip,	.972
22 lires,	1 pistole,	3.555
23 lires,	1 Spanish pistole,	3.61

ROME.

CIVITO-VECCHIA, ANCONA, &c.

5 quatrini make	1 bayoc,	$0.
8 bayocs,	1 julio,	.111
10 bayocs,	1 stampt julio,	.138
24 bayocs,	1 testoon,	.333
10 julios,	1 current crown,	1.111
12 julios,	1 stampt do.	1.333
18 julios,	1 chequin,	2.
31 julios,	1 pistole,	3.444

BOLONA, RAVENNA, &c.

6 quatrini make	1 bayoc,	$0.
10 bayocs,	1 julio,	.11
20 bayocs,	1 lire,	.22
3 julios,	1 testoon,	.333
85 bayocs,	1 scudi of exchange,	.944
105 bayocs,	1 ducatoon,	1.166
100 bayocs,	1 crown,	1.11
31 julios,	1 pistole,	3.444

NAPLES.

GAIETA, CAPUA.

3 quatrini make	1 grain, value in the U.S.	$0.
10 grains,	1 carlin,	.074
2 carlins, or 20 gr.	1 tarin,	.148
2 tarins, or 40,	1 testoon,	.296
2½ tarins, or 100,	1 ducat of exchange,	.74
23 tarins,	1 pistole,	3.407
25 tarins,	1 Spanish pistole,	3.70

SICILY AND MALTA.

PALERMO, SYRACUSE, MESSINA, AND VALETTO.

6 pichli make	1 grain, value in the U. S.	$0.
10 grains,	1 carlin,	.029
20 grains,	1 tarin,	.058
6 tarins,	1 florin of exchange,	.35
13 tarins,	1 ducat of exchange,	.73
60 carlins,	1 ounce,	1.71
2 ounces,	1 pistole,	3.41

GERMANY.

VIENNA, BLENHEIM, PRAGUE, BRESLAW, HUNGARY, FRANCONIA, &c. PRESBURG, FRANKFORT, NUREMBERG, &c.

2 fennings make	1 dreyer, value in the U. S.	$0.
4 fennings,	1 cruitzer,	
4 cruitzers,	1 batzen,	.084
15 batzen,	1 gould,	.517
90 cruitzers,	1 rix dollar,	.777
2 florins,	1 hard dollar,	1.
60 batzen,	1 ducat,	2.074

COLOGNE.

MENTZ, LIEGE, TRIERS, MUNICH, MUNSTER, PUDERBORN, &c.

3 dutes make	1 cruitzer, value in the U. S.	$0.
2 creutzers,	1 alb,	
8 dutes,	1 stiver,	.013
3 stivers,	1 palpert,	.039
4 palperts,	1 copstuck,	.155
40 stivers,	1 guilder,	.517
2 guilders,	1 hard dollar,	1.034
4 guilders,	1 ducat,	2.074

AFRICA.

MOROCCO, SANTACRUZ, MEQUINEZ, FEZ, &c.

24 fluce make	1 blanquil, value in U. S.	$0.037
3 blanquil,	1 armee,	.148
7 blanquil,	1 octavo,	.259
2 octavo,	1 quarto,	.518
2 quarto,	1 medio,	1.037
27 blanquil,	1 dollar,	1.
54 blanquil,	1 xequin,	2.
100 blanquil,	1 pistole,	3.72

BARBARY.

ALGIERS, TUNIS, TRIPOLI.

	1 asper. value in the U. S.	$0.0125
10 aspers make	1 rial,	.125
2 rials,	1 double,	.25
4 doubles,	1 dollar,	1.
24 medins,	1 chequin,	.75
32 medins,	1 dollar,	1.
180 aspers,	1 zequin,	1.96
15 doubles,	1 pistole,	3.72

EGYPT.

CAIRO, ALEXANDRIA, &c.

	1 asper, value in the U. S.	$0.0104
3 aspers make	1 medin,	.0212
24 medins,	1 Italian ducato,	.75
80 aspers,	1 piaster,	.888
32 medins,	1 dollar,	1.
96 aspers,	1 crown,	1.111
192 aspers,	1 sultanin,	2.222
72 medins,	1 Pargo dollar,	2.333

ASIA AND PERSIA.

ISPAHAN, ORMRIS, AND GOMBROON.

10 coz make	1 shahee, ualue in the U. S.	$0.074
4 shahees,	1 abashee,	.296
5 abashees,	1 or,	1.48
12 do.	1 bavelo,	3.555
50 do.	1 tomond,	14.81

ARABIA.

MEDINA, MECCA, AND MOCHA.

7 caret make	1 comashee, value in U.S.	$0.016
18 comashee,	1 abyss,	.30
60 do.	1 piaster, or dollar,	1.
100 do.	1 sequin,	1.666
9 sequin,	1 tomond,	15.

MOGOL. EAST-INDIES.

SURAY, CAMBRAY, GUZZURAY.

4 pieces make	1 fanam, value in U. S.	$0.034
4 fanam,	1 ana,	.138
4 anas,	1 rupee,	.555
2 rupees,	1 crown,	1.11
14 anas,	1 pagoda,	1.94
4 pagoda,	1 gold rupee.	7.77

MALABAR.

COMBAY, DUBAL, &c.

5 rez make	1 pice, value in U. S.	$0.00647
20 pices,	1 quarter,	.129
4 quarters,	1 rupee,	.518
14 do.	1 pagoda,	1.814
60 do.	1 gold rupee,	7.77

GOA, VISAPOUR, &c.

20 rez make	1 vintin, value in U. S.	$0.0231
42 vintin,	1 tangu,	.972
4 tangu,	1 paru,	3.888
2 paru,	1 gold rupee,	7.77

COROMANDEL.

MADRAS, PONDICHERRY, &c.

4 cash make	1 pice, value in U. S.	$0.0069
8 pices,	1 fanam,	.055
10 fanam,	1 rupee,	.555
2 rupees,	1 crown,	1.111
35 fanam,	1 pagoda,	1.944
4 pagoda,	1 gold rupee,	7.77

BENGAL.

CALCUTTA, FORT WILLIAM, &c.

4 pice make	1 fanam, value in U. S.	$0.0115
3 fanam,	1 ana,	.034
16 anas,	1 rupee,	.555
2 rupee,	1 crown,	1.11
56 anas,	1 pagoda,	1.94

SIAM.

PEGU, MALACCO, CAMBODIA, SUMATRA, JAVA, BORNEO, &c.

800 cori make	1 fettee, value in U. S.	$0.0011
125 fettee,	1 sataleer,	.138
250 fettee,	1 sooco,	.277
500 fettee,	1 tutal,	.555
900 fettee,	1 dollar,	1.
1000 fettee,	1 crown, or rial,	1.11

WEST-INDIES—ENGLISH.

JAMAICA, BARBADOES, &c.

4 farthings make	1 penny, value in the U.S.	$0.01⅓
12 pence,	1 shilling,	.16
20 shillings,	1 pound currency,	3.20
7½ pence,	1 bit,	.10
75 pence,	1 dollar,	1.
84 pence,	1 crown,	1.12
24 shillings,	1 pistole,	3.84
30 shillings,	1 guinea,	4.80

FRENCH.

ST. DOMINGO, MARTINICO, &c.

15 sols make	1 scalin.
20 sols,	1 livre.
7 livres,	1 dollar.
8 do.	1 ecu.
26 do.	1 pistole.
32 do.	1 louis d'or.

CHINA.

PEKIN, CANTON, &c.

10 caxa make	1 condareen, value in U.S.	$0.014
10 condareen,	1 mace,	.143
35 condareen,	1 rupee,	.50
2 rupees,	1 dollar,	1.
10 mace,	1 tale,	1.43

JAPAN.

	1 piti, value in the U S.	$0.0037
20 pitis make	1 mace,	.074
15 maces,	1 oz silver,	1.076
20 do.	1 tale,	1.481
30 do.	1 ingot,	2.22
13 oz silver,	1 oz gold,	14.
2 oz gold,	1 japanese,	28.
100 japanese,	1 cuttee,	280.

In Novascotia, Canada, Florida, Cayenne, &c. where English, French, and Spanish monies circulate, the currency alters according to the plenty or scarcity of specie.

For the currencies of the Spanish, Portuguese, Dutch, Danish, &c. in the West-Indies, see their respective exchanges.

For the information of the readers of history, the monies of the ancients shall be explained here.

JEWISH MONIES.

	1 gerah, valuein U. S.	$0.0253'
10 gerah make	1 bekah,	.253'
2 bekah,	1 shekel,	.506'
50 shekel,	1 mina,	25.333
60 mina,	1 talent,	1520.
1 sextula, gold,		2.704'
3 sextula,	1 siculus,	8.111'
3000 siculus,	1 talent gold,	24333.333'

GRECIAN MONIES.

	1 chalcus, value in U. S.	$0.006
2 chalcus make	1 dichalcus,	.011
2 hemibolum,	1 obolus,	.0239
2 obolus,	1 diabolum,	.047
2 diabolum,	1 tebrabolum,	.094
1½ tebrabolum,	1 drachma,	.143
100 drachma,	1 mina,	143.
60 mina,	1 talent,	861.

ROMAN MONIES.

	1 teruncius, val. in U.S.	$.0035
2 teruncius make	1 semilibella,	.0071
2 semilibella,	1 libella,	.0142
2½ libella,	1 sestertius,	.0357
2 sestertius,	1 victoriatus,	.0715
2 victoriatus,	1 demarius,	.143
1000 sestertius,	1 sestertium,	35.80
100 sestertia,	1 decies,	3580.

ARBITRATIONS OF EXCHANGE.

It is of the utmost consequence to the merchant, who has foreign concerns, to be well acquainted with the mode of arbitrating the exchanges between two or more places; to have a knowledge of their weights and measures, and the proportion they bear to each other. by this means, he may make his gain certain, his knowledge, as a merchant, respected; and may likewise acquire valuable correspondents abroad, that might, otherwise, never hear of his name.

To the Rule of Proportion belongs the solution of all questions in arbitration; but as continual statements are not only tedious, but liable to error, let the questions be solved by the following rule, called Conjoined Proportion.

1st. Distinguish the members of the arbitration into antecedents and consequents; placing the antecedents on the left, and the consequents on the right.

2d. The first consequent must be of the same name with the second antecedent, which order must be observed through the equation; and the last consequent must be of the same name with the first antecedent.

3d. If any of the terms in the equation have a fraction annexed, multiply the whole numbers by the denominator on the opposite side below. However, this may be dispensed with, by taking parts for the fraction, &c.

4th. Multiply the antecedents for a divisor, and the consequents for a dividend, if the place of the antecedent be blank; or, multiply the consequents for a divisor, and the antecedents for a dividend, if the place of the consequent be blank.

EXAMPLES.

1. Amsterdam owes Philadelphia 6000 guilders; whether is it better to draw at 37 cents the guilder, or have the money remitted by the following route, viz. To Paris, at 54*d.* Flemish, for 3 francs; thence to Genoa, at 5 francs per piaster; thence to London, at 50*d.* sterling, per piaster; and thence to Philadelphia, at par?

Ans. Gain by the remittance, $249.13.

Operation of the preceding.

18 $\cancel{54}$*d.* = $\cancel{3}$ francs.
$\cancel{5}$*fra.* = 1 piaster.
1 *pias.* = $\cancel{50}$*d.* ster. 10.
9 $\cancel{54}$*d.* = 1 dollar.
doll. $\cancel{24000}$*d.* Flemish. 40000.
18×9=1.62)40000(2469.13 by way of London.
6000×.37=2220

249.13 gain.

NOTE. By dividing crosswise, 3 into 3 and 54, 3 is stricken out, and 18 put for 54, which is also stricken out, and so of the rest; for the remainders are the same way proportional that the wholes are. Striking out on both sides is called cancelling.

The rule for abridging the antecedents and consequents is founded upon prop. 19. Euclid, book 5., which says, "If a whole magnitude, be to a whole, as a magnitude taken from the first, is to a magnitude taken from the other, the remainder shall be to the remainder as the whole to the whole."

The preceding question worked by proportion.

d. Flem. *d. Flem.* *fra.* *francs.*
First. 54 : 24000 :: 3 : 13333.3′
fr. *francs.* *pias.* *piasters.*
Second. 5 : 13333.3′ :: 1 : 2666.6′
pias. *piasters.* *d. ster.* *d. sterling.*
Third. 1 : 2666.6′ :: 50 : 133333.3′
d. ster. *d. sterling.*
Fourth. 54 : 133333.3′ :: 1 : $2469.13 as before; but the first method, on account of its brevity, is much preferable.

2. C, in London, owes D, in Philadelphia, £400 sterling; D orders the money to be remitted on his account to Hamburg, at 52*d.* sterling, per rix dollar; thence to Lyons, at 7 rix dollars for 13 crowns; thence to Venice, at 12 crowns for 7 ducats; thence to Malaga, at 7 ducats for 6 Spanish ducats of exchange; and thence to Philadelphia, at 110 cents per ducat, equal $59\frac{2}{5}$*d.* sterling. What will be his gain or loss, if the exchange between Philadelphia and London be at par?

52*d.* sterling = 1 rix dollar.
7 rix dollars = 13 crowns.
12 crowns = 7 ducats.
7 ducats = 6 Spanish ducats.
1 Spa. ducat = $59\frac{2}{5}$*d* sterl. = 110*cts.*

£400.

$$\frac{1\times\cancel{13}\times\cancel{7}\times\cancel{6}\times59\frac{2}{5}\times\overset{50}{\overset{\cancel{100}}{\cancel{400}}}}{\underset{\cancel{4}}{\cancel{52}}\times\cancel{7}\times\underset{\cancel{2}}{\cancel{12}}\times7\times1}=£424\tfrac{2}{7}\text{ sterling.}$$

$$\begin{array}{r} 400 \\ \hline £24\tfrac{2}{7} \end{array}=\$107.94\text{ Ans.}$$

By cancelling, $\frac{59\frac{2}{5}\times50}{7}=424\frac{2}{7}$, as before.

3. E, of Leghorn, is indebted to F, of Baltimore, 800 piasters; exchange between Baltimore and Leghorn, 99 cents per piaster; F considers it better to have the mo-

ney remitted, and E remits as follows: To Venice, at 94 piasters for 100 ducats; thence to Cadiz, at 320 mervadies per ducat; thence to Lisbon, at 630 reas for 272 mervadies; thence to Amsterdam, at 50*d*. Flemish for 400 reas; thence to Paris, at 56*d*. Flemish for 3 francs; thence to London, at 31½*d*. sterling for 3 francs; and from London to Baltimore, at 54*d*. sterling per dollar. What is the arbitrated price of a piaster between Baltimore and Leghorn, and what does F gain by the transaction? Ans. Arbitrated price, $1.02+, gain, $24.

4. G, of Venice, is indebted to H, of Philadelphia, 300 ducats, exchange on Venice at 90 cents per ducat; H considers it better to have the money remitted, and G remits as follows: To Barcelona, at 320 mervadies per ducat; thence to Oporto, at 420 mervadies for 1 milrea; thence to Nantz, at 140 reas per frank; thence to London, at 30*d*. sterling for 3 francs; and thence to Philadelphia, at par. What is the arbitrated price of a ducat, between Venice and Philadelphia, and what does H gain or lose by the transaction?

Ans. Arbitrated price for a dollar, $1.0078, gain, $32.54

CONCERNING THE GAIN AND LOSS BY EXCHANGE.

Suppose K, in London, owes L, in New-York, $1000; the exchange between New-York and London at 2 per cent above par, and the exchange between London and New-York at 52*d*. sterling per dollar; whether is it better for L to draw, or K to remit?

FOREIGN WEIGHTS AND MEASURES COMPARED WITH AMERICAN AND BRITISH.

If 6*lb*. of *a*, be equal to 5*lb*. of *b*, and 4*lb*. of *b*, equal 5*lb*. of c, and 8*lb*. of c, equal 9*lb*. of *d*, and 5*lb*. of *d*, equal 6*lb*. of e; how many *lb*. of e are equal to 40*lb*. of *a*?

$$\begin{array}{l} a6' = 5'b\\ b4' = 5c\\ c8 = 9d\\ d5' = 6'e\\ e = 40'a \end{array} \qquad \begin{array}{r} 10\\ 9\\ \hline 90\\ 5\\ \hline 8)450(56\tfrac{1}{4}lb.\ \text{Ans.}\\ 40\\ \hline 50\\ 48\\ \hline 2 \end{array}$$

Or, $\dfrac{\cancel{5}\times5\times9\times\cancel{6}\times\overset{5}{\cancel{40}}}{\cancel{6}\times4\times\cancel{8}\times\cancel{5}}=$

$\dfrac{5\times9\times5}{4}=56\tfrac{1}{4}lb.$ as before.

2. If 10*lb.* in London be equal to 9*lb.* at Amsterdam, and 45*lb.* at Amsterdam equal 49*lb.* at Bruges, and 98*lb.* at Bruges equal 116*lb.* at Dantzic; how many *lb.* at Dantzic are equal to 112*lb.* at London? Ans. 129.92*lb.*

3. If 109*lb.* at London, equal 100*lb.* at Amsterdam, and 60*lb.* at Amsterdam equal 90*lb.* at Genoa, and 100*lb.* at Genoa equal 70*lb.* at Leipsic, and 105*lb.* at Leipsic equal 105*lb.* at Leghorn, and 100*lb.* at Leghorn equal 109*lb.* at Seville, and 63*lb.* at Seville equal 100*lb.* at Milan; how many *lb.* at Milan will equal 63*lb.* at London?

Ans. 105*lb.*

4. What will 1*lb.* of pepper cost, if 3*lb.* of cloves cost as much as 6*lb.* of pepper, and 2½*lb.* cinnamon cost as much as 4*lb.* of cloves, and 3*lb.* of cinnamon cost 8 shillings?

Ans. 10*d.*

5. If 7 aunes of Genoa, make 9 yards of London, and 36 yards of London equal 49 aunes of Holland, and 7 aunes of Holland equal 9 braces of Milan, and 3 braces of Milan equal 2 vares of Aragon, and 5 vares of Aragon equal 2 canes of Montpelier, 9 canes of Montpelier equal 10 canes of Thoulouse, and 4 canes of Thoulouse equal 9 aunes of Troyes; how many aunes of Troyes will be equal to 100 aunes of Geneva? Ans. 150 aunes of Troyes.

Solution 1.

$$\frac{9\times49\times9\times2\times2\times10\times9\times100}{7\times36\times7\times3\times5\times9\times4}=150 \text{ aunes Troyes.}$$

Solution 2.

$$\begin{array}{r}
\cancel{7}=\cancel{9} \\
\cancel{2}\ \cancel{4}\ \cancel{36}=\cancel{49}\ \cancel{7} \\
\cancel{7}=\cancel{9} \\
\cancel{3}=\cancel{2} \\
\cancel{5}=\cancel{2} \\
\cancel{9}=\cancel{10}\ 2 \\
\cancel{4}=\cancel{9}\ 3 \\
\cancel{100} \\
25 \\
3 \\
\hline
75 \\
2 \\
\hline
150 \text{ Ans.}
\end{array}$$

Here it may be observed, that if equals be struck from both sides, the remainders being worked with according to rule, will give the answer the same as the wholes would have done.

TABLE,

SHOWING HOW MANY POUNDS AND DECIMALS OF POUNDS AVOIRDUPOISE ARE EQUAL TO 100*lb.* IN THE FOLLOWING PLACES.

Comparative tables of weights and measures, for the most part, give 100*lb.* of London, Amsterdam, or Paris, as the standard to which all the rest are made equal; but it will be found to facilitate calculation, by making 100*lb.* foreign the standard, and show how many *lb.* avoirdupoise are equal thereto.

Foreign.	American. English.
HOLLAND.	
Equals.	
100*lb.* at Amsterdam Dort,	109*lb.*
Leyden,	102.8
Rotterdam,	109.
BRABANT.	
Bergen Opzome,	112.3
Antwerp,	103.8
100*lb.* at Arschot,	*lb.* 103.8
Bois le Duc,	103.8
Lovain,	103.8
DENMARK.	
Copenhagen,	101.46
Hamburg,	106.8
Lubec,	103.8

FRANCE.

Foreign. Equals,	American. English.
100*lb.* at Avignon, *lb.*	90.75
Bayone,	109.
Bisancon,	109.
Bourdeaux,	109.
Bourgen bress,	104.8
Lyons, city *wt.*	94.
silk *wt.*	101.5
Marlines,	103.8
Marseilles,	88.2
Montpelier,	91.
Nantz,	109.
Nancy,	103.
Paris,	109.
Rochelle,	109.
Rouen,	113.5
St. Maloes,	109.
Thoulouse,	135.

GERMANY.

100*lb.* at Liege,	103.8
Lisle,	96.
Nuremberg,	111.22
Nauremberg,	104.3
Stetin,	99.
Bremen,	105.81
Breslaw,	87.1
Bruges,	103.8
Erford,	100.25
Frankfort, on the main,	111.22
Frankfort, on the Oder,	111.22
Ghent,	103.8
Leipsic,	103.8

PRUSSIA.

100*lb.* Koningsberg,	87.1

SPAIN.

Foreign. Equals,	American. English.
100*lb.* at Alicant, *lb.*	100.9
Bilboa,	109.
Cadiz,	103.8
Madrid,	96.
St. Sebastian,	109.
Saragosa,	69.
Seville,	102.8
Valentia,	69.

ITALY.

Bergamo, sm. *wt.*	65.46
great do.	160.3
Bolona,	72.
Florence, sm. *wt.*	76.2
great do.	117.2
Genoa, cash do.	67.6
common do.	109.
great do.	120.4
Leghorn, sm. do.	76.07
great do.	117.2
Lucca, small do.	76.8
great do.	115.34
Messina,	73.85
Milan,	65.2
Naples,	64.3
Palermo, sm. *wt.*	71.
great do.	154.3
Rome,	75.
Venice, sm. do.	60.15
great do.	114.13
Piedmont,	72.
Turin,	72.
Savoy,	122.8

PORTUGAL.

100*lb.* Lisbon, &c.	102.8

RUSSIA. Foreign.	American. English. Equals,	POLAND. Foreign.	American. English. Equals,
100*lb.* Archangel,	*lb.* 90.75	100*lb.* Dantzic,	*lb.* 96.
Muscovy,	87.14	Elbing,	96.5
Revel,	96.83	Koningsberg,	87.14
Riga,	100.		

By this table, the proportion of 1*lb.* may be known as readily as 100, by removing the decimal point two places further to the left hand: thus, 100*lb.* at Genoa is equal to 109*lb.* at New-York; and 1*lb.* in Genoa, according to the same proportion, is equal to 1.09*lb.* in New-York.

PARTICULAR WEIGHTS OF FOREIGN NATIONS.

SPANISH WEIGHT COMPARED WITH AVOIRDUPOISE.

			lb.	*oz.*
16 adarme,	1 ounce,	equals	0	0.96
16 ounces	1 pound,			15.36
25 pound	1 aroba,		24	
4 aroba,	1 quintal,		96	
6 do.	1		144	

GOLD WEIGHT COMPARED WITH TROY WEIGHT.

			lb.	*oz.*	*dwt.*	*gr.*
1 tomin		equal	0	0	0	8.29
8 do.	1 castilian,		0	0	2	19.16
100 castilian,			1	1	19	20.16

WEIGHTS OF PORTUGAL COMPARED WITH AVOIRDUPOISE.

		lb.	*oz.*
1 arratal, or pound,		1	00.368
2 pound,	1 faratella,	2	00.736
12 do.	1 rottoli,	12	4.416
32 do.	1 aroba,	32	11.776

WEIGHTS OF ITALY COMPARED WITH AVOIRDUPOISE.

		lb.	oz.
1 pound,		1	2.26
30 do.	1 mirre,	44	3.82
4 mirre,	1 migliaro,	136	15.29
	Rotollo of Genoa,	24	

IN GERMANY, FLANDERS, HOLLAND, SWEDEN, DENMARK, &c.

One schippondt is equal to

		lb.	oz.	
300*lb.*	at Antwerp,	311	6.4	Avoirdupoise.
300	Hamburg,	329	6.4	
320	Lubeck,	332	2.56	
400	Konigsberg,	348	9.	
320	Sweden, for copper,	324	10.72	
400	do. provisions,	405	13.44	
400	Riga,	400		
400	Revel,	387	5.12	
340	Dantzic,	326	6.4	
300	Norway,	341	11.2	
300	Amsterdam,	341		
The	lyspondt of Amsterdam, and is	16	5.6	

WEIGHTS OF TURKEY. GREAT WEIGHT.

		lb.	oz.	
1 occo	is equal to	3	12.8	Avoirdupoise.
6 do.	1 batman,	32	12.8	
44 do.	1 quintal,	167	3.2	

SMALL WEIGHT.

			lb.	oz.
1 occo,	is equal to		0	15 Avoirdu.
6 do.	1 batman,		5	10
1 rottoli of Egypt,	144 drams,		1*lb.*	6.4*oz.*
1 do. large (Aleppo)	720	do.	7	
1 do. small do.	624	do.	6	1.22
1 do. Scyda,	600	do.	5	13.28

WEIGHTS OF MUSCOVY.

		lb.	oz.	
1 poede	is equal to	8.	11.42	Avoir.
10 do.	1 berkewitz,	87	3.24	

MEASURES OF FOREIGN NATIONS, COMPARED WITH INCHES, &c.

FRANCE.

	Foreign.	Inches.
	Metre,	39.383
Aune of	Abbeville,	42.
	Bayone,	
	Bourdeaux,	
	Caen,	
	Calais,	
	Elberuf,	
	Havre deGrace	
	Nantz,	
	Paris,	
	St. Quintins,	
	Rochelle,	
	Rouen,	
	Sedan,	
	Cambray,	38.8
	Lyons,	44.5

BRABANT.

	Foreign.	Inches.
Aune of	Abo,	27.4
	BergenOpzome	
	Bois le Duc,	
	Breda,	
	Brussels,	

HOLLAND.

	Foreign.	Inches.
Aune of	Amsterdam,	27.
	Campvier,	
	Hague,	
	Haerlem,	
	Leyden,	
	Rotterdam,	
	Flushing,	
	Middleburg,	
	Gueldres	26.1

FLANDERS.

	Foreign.	Inches.
Aune of	Arras,	27.7
	Bruges,	26.3
	St. Omers,	
	Ostend,	
	Lisle,	28.8
	Douay,	28.
	Ypres,	
	Dunkirk,	26.6
	Ghent,	
	Namur,	26.1
	Tournay,	24.
	Valenciennes,	26.

PORTUGAL.

Foreign.	Inches.
Barras of Lisbon,	44.3
Cavido of do.	27.

SPAIN.

	Foreign.		Inches.
Barras, cont'g 2½ palmos,	Almeria,		27.4
	St Andero,		33.3
	Bilboa,		
	St. Sebastian,		
	Cadiz,	silk,	32.4
		cloth,	33.8
	Carthagena,		32.4
	Saley,		
	Coruna,		33.1
	Madrid,		
Aune of	Cadiz,		27.4
	Alicant,		37.6
	Barcelona,		63.
	Saragosa,		71.
	Tortosa,		66.
	Valentia,		37.6

SPANISH ISLANDS.

Foreign.	Place		Inches.
Camas of 8 Palmas or 4 Brazzes	Minorca,		63.
	Majorca,		37.8

ITALY.

Foreign.	Place		Inches.
Aune of Geneva,			44.5
Ell of	Ravenna,		26.5
	Trent,		21.6
Camas of	Capua, Gaieta, Messina, Naples, St. Remo,		88.2
	Salermo, Savona,		88.2
	Rome, Civita,		81.4
	Florence,		73.
	Genoa,	woolen,	88.3
		linen,	98.1
	Palermo, Syracuse	Sicily,	88.2
	Malta,		88.2
Brazzas,	Bergama, Bolona,		26.5
	Rome, Farara,		36.
	Florence, Luca,		23.4
	Genoa,		22.8
	Parma, Mantua, Modena, Placentia,		26.8
	Milan,	silk,	20.9
		cloth,	26.5
	Venice,	silk,	22.5
		cloth,	26.5

BAVARIA.

Foreign.	Place		Inches.
Ell of	Munich,		21.2
	Ratisbon, Saltzburg		21.6

SWITZERLAND.

Foreign.	Place		Inches.
Ell of	Bazil, Berne,		22.7
	St. Gall	woolen,	24.1
		linen,	31.3

BRABANT.

Foreign.	Place		Inches.
Aune of	Louvain, Maestreicht, Malines, Narva, Strasburg,		27.4
	Antwerp,		27.6

GERMANY.

Foreign.	Place		Inches.
Ell of	Aixlachapelle, Dusseldorf, Mentz,		21.6
	Augsburg,	linen,	35.
		woolen,	31.7
	Bonn, Coblentz, Cologn, Manheim, Philipsburg, Triers,		22.
	Nuremberg,		27.
	Osnaburg,		44.7
Aune of Friburg,			30.6

SAXONY.

Foreign.	Place		Inches.
Ell of	Bremen, Leipsic, Nauremburg,		23.
	Dresden, Wesmar,		21.2

SILESIA.

Foreign.	Place		Inches.
Ell of Breslaw,			21.6

AUSTRIA.

Foreign.	Place		Inches.
Ell of	Vienna,		22.
	Inspruck, Bolsano,		21.6

PRUSSIA.

Foreign.	Place		Inches.
Ell of Berlin,			23.4
	Stetin,		21.6

Foreign.	Inches.
BOHEMIA.	
Ell of Prague,	21.6
BRANDENBURG.	
Ell of Potsdam,	23.4
LOW COUNTRIES.	
Aune of Nimeguen,	26.1
Liege,	24.1
Ruremond,	27.
HAINAULT.	
Aune of Mons,	25.2
FRANCONIA.	
Ell of Frankfort,	22.3
DENMARK.	
Ell of Copenhagen,	24.3
Kiel,	22.
Altona, Lubec,	22.4
SWEDEN.	
Ell of Abo, Christianople,	23.4
NORWAY.	
Ell of Bergen, Christigna, Drontheim,	24.3
POLAND.	
Ell of Warsaw, Cracow,	21.6
Thorn,	23.
Dantzic,	24.
POMERANIA.	
Ell of Embden, Paderborn,	21.2
Munster,	21.6
SYRIA.	
Pico of Aleppo.	24.
Damascus,	27.
Scanderoon,	24.

Foreign.		Inches.
HANOVER.		
Ell of Zell,		21.6
RUSSIA.		
Arsheen of 16 vesh: Archangel, Narva, Petersburg, Moscow, Mitaw,		25.6
GREECE.		
Pico of Athens, Lacedemon,		18.
LESSER ASIA.		
Pico of Ephesus, Smyrna,		24.
ROMANIA.		
Pico of Adrianople,		27.
Galiopoli,		18.
PALESTINE.		
Pico of Jerusalem, Acre,		24.
ARABIA.		
Pico of Bassora,		24.
Modina, Mecca, Suez,		27.
Cavido of Mocko, Goa,		27.
AFRICA, EGYPT.		
Cavido of Alexandria, Sayde,		24.
Tunis & Tripoli,	cloth,	27.
	linen,	24.
	silk,	18.
Oran,		27.
Algiers,		27.
Barcu,	cloth,	24.
	silk,	18.

ALLIGATION.*

Alligation is a rule, by which two or more simples, of different qualities, may be mixed together, so that the composition may be of a mean or middle quality.

When the quantities and prices of several things are given, to find the mean price of the mixture composed of those things, it is called

ALLIGATION MEDIAL.

In Medial Alligation, the sum or total of all the quantities is in proportion to any part of them, as their total price, is to the price of any part of them; which is a mean medial, or middle price.

RULE.

Multiply any given part of the composition by the value of the whole, divide by the sum of said quantities, and the quotient will be the mean price, or value.

EXAMPLES.

1. A grocer had brown sugars, at 9, 10, 11, and 12 dollars per *Cwt.*; now suppose he would mix them together, at what price could he sell them, so as neither to lose nor gain?

*C.*1 at 9*D.*
1 at 10
1 at 11
1 at 12
—— *C.* *D.cts.*
4 : 42 :: 1 : 10.50 Ans.
C. *C.* *D.* *D.cts.*
More properly 4 : 1 :: 42 : 10.50 Ans.

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* Derived from the Latin (*ad*, to, and *ligo*, to bind) to tie, or bind together; it being necessary, in sundry instances, to tie or link the given quantities together, in order to perform the calculations.

2. A merchant mixed 50 gallons of rye whiskey, at 30*cts.* per gallon, with 40 gallons, at 32*cts.* and 60 gallons at 32½*cts.* per gallon; what should a gallon of the mixture be worth? Ans. 31$\frac{8}{15}$*cts.*

3. A vintner mixed 40 gallons of sack, worth $1 per gallon, 24 gallons of canary, at 87½*cts.* per gallon, 30 gallons of sherry, at 67*cts.* per gallon, and 10 gallons of white wine, at 60*cts.* per gallon; what is a gallon of the mixture worth? Ans. 83¾*cts.*

4. An oil and colour-man bought 30*Cwt.* of white lead, at $10 per *Cwt.* 27*Cwt.*, at $11, 18*Cwt.*, at $9 and 17*Cwt.*, at $13 per *Cwt.*; now, suppose he grinds the good and bad together, and sells it at 10 per cent advance, what is the selling price?

Ans. $11.71⅔+

ALLIGATION ALTERNATE.*

Alligation Alternate, is a method, when the rates of several commodities are given, to find such quantities of them as, being mixed together, shall bear a price propounded.

RULE.

1. The prices, or rates, must all be of the same denomination, (if not so given, must be reduced to the same name.)

2. Set down the prices, one above the other, and the mixed, or mean rate, on the left hand of these.

3. Join, or link together, the several rates, so that every one less than the mean, be linked with some one greater, or with as many as you please greater, and each of the greater with one less, or as many less as you please.

4. Take the difference between the mean rate and each price, and set them alternately. If only one

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* So called on account of the differences between the mean rate and of each other rate being alternately placed for sake of preserving equality.

difference stand against any rate, it will be the quantity belonging to that rate; but if there are several, then their sum will be the quantity, which sum will be the answer for that rate against which it stands.

EXAMPLES.

1. A grocer would mix sugars, at $9, $11, and $13 per *Cwt.*, so that the mixture should stand him $12 per *Cwt.*; what quantity of each sort must he take?

12	9	1*C.* at 9	For 9×1= 9
	11	1*C.* at 11	11×1=11
	13	3+1=4*C.* at 13	13×4=52

6)72(12proof

2. A merchant would mix wines, at $1.20, 1.50, 2, and 2.50, per gallon, so that the mixture should stand him in $.75 the gallon; what quantity of each sort must he take?

175	120	75	or 15	or 3
	150	25	5	1
	200	25	5	1
	250	55	11	2½

175	120	25	or 5	or 1
	150	75	15	3
	200	55	11	2½
	250	25	5	1

NOTE. Any number of answers may be had, by dividing, or multiplying, the first set of differences by any common divisor or multiplier; which is evident from the annexed examples.

NOTE 2. By connecting the less rate with the greater, and placing the difference between them and the mean rate alternately, the quantities, or differences, resulting, are such, that there is precisely as much gained by one quantity, as is lost by the other; therefore, the gain and loss, upon the whole, are equal, and exactly the proposed rate. In like man-

3. A miller purchased wheat for $1, 1.25, 1.50, and $2 per bushel; but is minded to mix such proportions of them together as will bear a price of $1.37½ cents per bushel; how much of each sort must be taken to make the required mixture?

Ans. 5 bus. at $1, 1 bus. at 1.25, 1 bus. at 1.50, and 3 bus. at 2.00.

4. A goldsmith desired to mix gold of 19 carats fine, with some of 16, 18, 23, and 24 carats fine, so that the compound may be 21 carats fine; what quantity of each must he take?

Ans. 3 of 16, 2 of 18, 2 of 19, 5 of 23, and 5 of 24. But this admits of many other differences.

ALLIGATION PARTIAL.*

Alligation Partial is a method, when the price of each simple is given, also the quantity of one of them, and the mean rate, to find the several quantities of the rest, in proportion to that given.

RULE.

1. Take the difference between each price and the mean rate.

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ner, let the number of simples be what they may, and with how many soever each one is linked, since it is always a less with a greater than the mean price, there will be an equal balance of loss and gain between every two, and consequently an equal balance in the whole; which might be easily demonstrated, if necessary. If the reader should fancy to see a complete demonstration of this rule, let him turn to page 231, prob. 79, Emerson's Algebra, and there he will find one.

* Alligation Partial, is so called, because it falls short of being general; the rule only applying in particular cases, or in limited or restricted cases.

2. Then say, as the difference of that simple, whose quantity is given, : is to the known quantity, : : so is any other difference, : to the quantity of its opposite name.

EXAMPLES.

1. How much sugar, at 9, 10, 11, and 12 cents per *lb.* must be mixed with 50*lb.*, at 20 cents per *lb.* to be worth 12½ cents per *lb.*?

$$12\tfrac{1}{2}\left|\begin{array}{l|l} 9 & 7\tfrac{1}{2} \\ 10 & 7\tfrac{1}{2} \\ 11 & 7\tfrac{1}{2} \\ 12 & 7\tfrac{1}{2} \\ 20 & 3\tfrac{1}{2}+2\tfrac{1}{2}+1\tfrac{1}{2}+\tfrac{1}{2}=8 \end{array}\right.$$

$$\left.\begin{array}{l} 8 : 7\tfrac{1}{2} :: 50 : 46\tfrac{7}{8} \\ 8 : 7\tfrac{1}{2} :: 50 : 46\tfrac{7}{8} \\ 8 : 7\tfrac{1}{2} :: 50 : 46\tfrac{7}{8} \\ 8 : 7\tfrac{1}{2} :: 50 : 46\tfrac{7}{8} \end{array}\right\}\text{Or an equal quantity of each.}$$

Proof of the above as follows, viz.

$$46\tfrac{7}{8}\times\left\{\begin{array}{ll} 9=421\tfrac{7}{8}lb. \text{ at} & 9cts. \\ 10=468\tfrac{6}{8}lb. & 10cts. \\ 11=515\tfrac{5}{8}lb. & 11cts. \\ 12=562\tfrac{4}{8}lb. & 12cts. \end{array}\right\}\text{with 50 at 20 to make a mixture worth } 12\tfrac{1}{2}cts. \text{ per } lb.$$

1968.75

50 × 20 = 1000.

46⅞ × 4 + 50 = 237.5)2968.75(12.5 Proof.

```
2375
----
 5937
 4750
 -----
 11875
 11875
 -----
     0
```

2. How much malaga, at $1.12½ per gallon, sherry, at 87½, white wine, at 62½, must be mixed with 30 gallons of canary, at 75 cents per gallon, so that the mixture may stand in 68¾ cents per gallon?

Ans. 30 gal. at 112½, 30 at 87½, and 330 at 62½ cents per gallon.

3. How much alloy, and how much gold, of 21, and 23 carats fine, must be put to 30 ounces, of 20 carats fine, to bring it to 18 carats fine?

Ans.

ALLIGATION TOTAL.*

Alligation total is a method, when the particular rates of all the ingredients proposed to be mixed, the sum of all their quantities, with the mean rate of that sum being given, to find the particular quantities of the mixture.

RULE.

Set down all the particular rates, with the mean rate, as before; find the differences, add all the differences into a sum; then say, as the sum of all the differences : is to the sum of all the quantities given, : : so is each particular difference, : to its particular quantity required.

EXAMPLES.

1. A grocer has three kinds of rum; one at 50, one at 60, and one at 70 cents per gallon, and has a mind to put up a cask of 30 gallons of the three mixed, so

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* Alligation Total, so called, on account of so much being given in the problems belonging to this ease. In fact, the total, or whole matters are given, to find the particular quantities of the mixture, or of which it is composed.

that it may be worth just 56¼ cents per gallon; how much of each sort must he take?

$$56\tfrac{1}{4}\left\{\begin{array}{l}50\\60\\70\end{array}\right.\begin{array}{l}13\tfrac{3}{4}+3\tfrac{3}{4}=17\tfrac{1}{2}\\6\tfrac{1}{4}\\6\tfrac{1}{4}\end{array}\left.\vphantom{\begin{array}{l}1\\1\\1\end{array}}\right\}=\begin{array}{r}\overline{17\tfrac{1}{2}+6\tfrac{1}{4}+6\tfrac{1}{4}}:30::17\tfrac{1}{2}:17\tfrac{1}{2}\\30:30::6\tfrac{1}{4}:6\tfrac{1}{4}\\30:30::6\tfrac{1}{4}:6\tfrac{1}{4}\end{array}$$

$$\begin{array}{ll}\text{Proof.} & 17\tfrac{1}{2}\times .50=8.75\\ & 6\tfrac{1}{4}\times .60=3.75\\ & 6\tfrac{1}{4}\times .70=4.375\\ & \overline{}\end{array}$$

Stated quantity, 30)16.875(56.25*cts.* stated price.

2. A druggist has simples, at 18, 15, 12, 9, 8, and 5 cents per *lb.*; how much of each sort must he take, to make up 318*lb.* worth 11*cts.* per *lb.*

$$\text{Ans.}\left\{\begin{array}{ll}82\tfrac{22}{23} & \text{at } 18\textit{cts.}\text{ per }\textit{lb.}\\43\tfrac{11}{23} & 15\\27\tfrac{16}{23} & 12\\13\tfrac{19}{23} & 9\\55\tfrac{7}{23} & 8\\96\tfrac{18}{23} & 5\end{array}\right.$$

3. Hiero, king of Syracuse, gave orders for a crown to be made, entirely of pure gold; but suspecting the workmen had debased it, by mixing with it silver or copper, he recommended the discovery of the fraud to the famous Archimedes, and desired to know the exact quantity of alloy in the crown.

Archimedes, in order to detect the imposition, procured two other masses, the one of pure gold, and the other of silver or copper, and each of the same weight with the former; and by putting each separately into a vessel full of water; the quantity of water expelled by them, determined their specific bulks. From which, and their given weights, it is easier to determine the quantities of gold and alloy in the crown by this case of Alligation, than by an algebraic process.

Suppose the weight of each mass to have been 5*lb.*, the weight of the water expelled by the alloy, 23 ounces, by the gold, 13 ounces, and by the crown, 16 ounces; that is, that their specific bulks were as

23, 13, and 16; then what were the quantities of gold and alloy in the crown.

$$16\left|\begin{array}{l}13\\23\end{array}\right)\begin{array}{l}7 \text{ of gold,}\\3 \text{ of alloy.}\end{array}\Bigg\}$$ And the sum of these is $7+3=10$, which should have been but 5.

Wherefore by the rule,

$$10 : 5 :: \left\{\begin{array}{l}7 : 3\tfrac{1}{2}lb. \text{ of gold.}\\3 : 1\tfrac{1}{2}lb. \text{ of alloy.}\end{array}\right\} \text{Ans.}$$

4. A cask of 58 gallons is filled with liquor of 7, 8, and 10 cents the gallon, and then it stands in $9\frac{1}{4}$*cts.* the gallon; I would know how many gallons of each sort was taken. Ans. $40\frac{6}{10}$ gal. at 10*cts.* $8\frac{7}{10}$ gal. at 8*cts.* and $8\frac{7}{10}$ at 7*cts.*

* * *

SINGLE POSITION,

OR THE RULE OF FALSE,

Is so called, because by supposed numbers, taken at adventure, and worked with according to the nature of the question, the true number sought is discovered.

Take any number, and perform the same operations with it, as are described to be performed in the question, by the following

RULE.*

As the sum total of the errors,
Is to the given sum,
So is the supposed number,
To the true one required.

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* If we put s=sum total of errors, S=given sum, n=supposed number, and N=true number; then by the nature of the rule, $s : S :: n : \frac{Sn}{s}=N$ Ans.

And by the resolution of equations, $s=\frac{Sn}{N}$. $S=\frac{sN}{n}$. $n=\frac{sN}{S}$. $N=\frac{Sn}{s}$.

EXAMPLES,

1. A teacher said that the $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of his scholars amounted to 87; how many students had he?

Suppose he had 1 scholar, then, per question,

$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$ of $1=\frac{29}{20}$, therefore,

$\frac{29}{20} : 1 :: 87 : 60$ Ans.

From this operation it appears, that the result of the position, bears the same proportion to the *position*, that the given number does to the *number required*, which is a principle to be observed throughout the rest of this case.

2. Four merchants, A, B, C, and D, have gained $1000, which they divide in manner following, viz. That half the share of A, is severally equal to $\frac{3}{4}$ the share of B, $\frac{5}{6}$ the share of C, wanting $11, and $\frac{6}{7}$ the share of D, plus $11; required the share of each.

Ans. $\left\{\begin{array}{l}\text{A's } 350\frac{50}{57}. \\ \text{B's } 233\frac{157}{171}. \\ \text{C's } 221\frac{30}{57}. \\ \text{D's } 193\frac{116}{171}.\end{array}\right.$

$1000 proof.

3. A company of students drank at a reckoning a certain sum, and said that $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{9}$ of the money made exactly 17\frac{8}{9}$; required, their reckoning.

Ans. $20.

4. A man dying, left $10000 to be divided amongst his three sons, (whose ages were 19, 16, and 10 years respectively) in such a manner that their several portions, when they became of age, might be equal, reckoning simple interest at 6 per cent, during their minorities; required, the share of each.

Ans. $\left\{\begin{array}{l}\$3942.849+ \\ 3396.916+ \\ 2660.235+\end{array}\right.$

DOUBLE POSITION.

Double Position is a rule which requires, first, that a number be chosen at random, and worked with as if the true one, and the error noted, which is called the first error, (and is found by comparing the result of the position with the given number.)

2. A second number must also be chosen, and worked with as before, and the error also noted.

3. Place down the positions, one above the other, the error of the second opposite the first position, and that of the first, opposite to the second.

4. If the errors are like, that is, both greater, or both less,

RULE.

1. Multiply each position by the error which stands opposite to it, and the difference of the products, divided by the difference of the errors, will give the number required.

2. If the errors be unlike, that is, one greater and the other less, the sum of the products must be divided by the sum of the errors, and the answer is had, as before.

EXAMPLE.

Let $100 be so divided amongst A, B, and C, that C may have $5 more than B, and B $5 more than A.

First, suppose A's share=1
B's 6
C's 11
18 and 100—18=82=first error, too small.

Again, suppose A's share=2
B's 7
C's 12
21 and 100—21=79=second error, too small.

Hence, by the first condition of the rule,

$$\left.\begin{array}{lll} \text{1st position } 1 & 79 & \text{2d error.} \\ \text{2d position } 2 & 82 & \text{1st error.} \end{array}\right\}$$ These are like errors, whether both be too small or too great.

$$\text{Therefore, } \frac{\overline{2\times82}-\overline{1\times79}}{82-79}=\$28\tfrac{1}{3}=\text{A's share.}$$

$$\begin{array}{r} 33\tfrac{1}{3}=\text{B's} \\ 38\tfrac{1}{3}=\text{C's} \\ \hline \$100.\ \text{Proof.} \end{array}$$

But suppose the second position had been 30.

$$\begin{array}{l} \text{Then } 30 \\ \quad\ \ 35 \\ \quad\ \ 40 \\ \hline \quad 105 \text{ and } 105-100=5=\text{2d error, too great.} \end{array}$$

Then by the second condition of the rule,

$$\begin{array}{ll} \text{P.} & \text{E.} \\ 1 & 5 \\ 30 & 82 \end{array}\Bigg\}$$ These errors are unlike, because one is too great and the other too small.

$$\text{Therefore, } \frac{\overline{30\times82}+\overline{1\times5}}{82+5}=\left.\begin{array}{r} \$28\tfrac{1}{3}=\text{A's.} \\ 33\tfrac{1}{3}=\text{B's.} \\ 28\tfrac{1}{3}=\text{C's.} \end{array}\right\}\text{as before}$$

$$\begin{array}{r} \hline 100.\ \text{Proof.} \end{array}$$

Remark. The first corrollary, deduced from the demonstration below, is the rule just now made use of. Corrollary second, will shew a different method of doing the same thing.

DEMONSTRATION.

If the numbers A and B, be produced from *a* and *b*, by any similar operation, to find the number from which N is produced, by a like operation; supposing the numbers A, B, N, to be as the differences of *a*, *b*, and the unknown number;

EXAMPLES.

1. If A, B, and C, own amongst them a ship, worth $12000; what is the proper share of each, when B owns a part of her worth $1000 more than that of A, and C $2000 worth more than B?

$$\text{Ans.}\begin{cases}\text{A's share} & \$2666\frac{2}{3}.\\ \text{B's} & 3666\frac{2}{3}.\\ \text{C's} & 5666\frac{2}{3}.\end{cases}$$

Let z = the number sought, and put the differences $\underline{\overset{a}{A}\ \overset{b}{B}\ \overset{z}{N}}$ $N-A=r$, $N-B=s$. Then by the question, $r : s :: z-a : z-b$ and $rz-rb=sz-sa$, by multiplying means and extremes. And by transposition, $rz-sz=rb-sa$, and $z=\frac{rb-sa}{a-s}$; if s be negative (or B greater than N), then $z=\frac{rb+sa}{r+s}$ the number sought.

Corrollary 1. If both A and B be less than N, or both greater, then $z=\frac{rb-sa}{r-s}$. But if only one, as B, be greater than N, then s is negative, and $z=\frac{rb+sa}{r+s}$

That is, if each supposed number be multiplied by the error of the other, and the difference of the products be divided by the difference of the errors, when the errors are like; or the sum of the products divided by the sum of the errors, when the errors are unlike; the quotient gives the number sought.

Corrollary 2. Hence also is derived another method.

Multiply the difference of the supposed numbers, by the least error, and divide the product by the difference of the errors, if like; or by the sum, if unlike; the quotient is the correction of the number

2. The sum of two numbers is 30, and if you compare the sum of $\frac{1}{2}$, $\frac{1}{3}$ of $\frac{1}{6}$ of the greater, with $\frac{1}{2}$, $\frac{1}{4}$ of $\frac{3}{4}$ of the lesser, you will find a perfect equality; required the numbers. Ans. 18 and 12.

3. The sum of two numbers is 50; now, if you divide the greater part by 7, and multiply the lesser by 3, the sum of the quotient and product will be equal to that given; required the parts.

Ans. 35 and 15.

4. A young gentleman having asked his father how old he was, received the following reply: seven years ago, my age was in a three-fold ratio to yours; but, if we should both happen to live seven years hence, my age shall be just double to yours. I desire to know their several ages.

Ans. 49 and 21 years.

5. A gentleman and a lady discoursing of their ages, observed, that 20 years ago, her age was to his in the ratio of 6 to 7, and that if they should happen to live together 20 years longer, her age should then be to his in the ratio of 14 to 15. I desire to know their respective ages.

Ans. 50 and 55 years respectively.

6. A drover being asked how many cattle he had, answered, that $\frac{1}{9}$ of them, plus 10, was just equal to $\frac{1}{7}$ of them, minus 10. Required, the number.

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belonging to the least error, which correction is to be added or subtracted, according as that number was too little or too great.

For, let s be the least error, being the error of b, and q the correction; then if A, B be less than N, $b+q=z$ and $q=z-b=\frac{rb-sa}{r-s}-b=\frac{b-a}{r-s}\times s$. But if B is greater than N, then $b-q=z$, and $q=b-z=b-\frac{rb+sa}{r+s}=\frac{b-a}{r+s}\times s$.

In order to shew how questions in Double Position may be resolved, where the ratios of the quantities are given, let question 4th be taken as an example.

First. Suppose 3 for the father's age, 7 years ago,
Then will 1 be the son's;
For 3 : 1 in the given ratio at that time,
By condition 2d, 3+14 : 1+14 :: 2 : $\frac{30}{17}$; but should be only 1;
Therefore, 30—17=13=first error.

Again. Suppose 6 for the father's age, 7 years ago, then will 2=the son's;
For 6 : 2 in the given ratio at that time.
By condition 2d, 6+14 : 3+14 :: 2 : $\frac{32}{20}$; but should be only 1;
Therefore, 32—20=12=second error.

Lastly, $\overline{13-12} : 1 :: \overline{13\times6-12\times3} : 42$=father's age at first, and $\frac{42}{3}$=14=son's age at first; For 42 : 14 :: 3 : 1, and 42+14 : 14+14 :: 2 : 1; but 42+7=49=father's at the time they conversed, and 14+7=21=son's age.

EXAMPLE 2d, OF SOLUTION.

Question 5th.

First. Suppose 3 and 3.5 for their ages 20 years ago; for 3 : 3.5 :: 6 : 7, the given ratio at that time.
By condition 2d, 3+40 : 3.5+40 :: 14 : $\frac{609}{43}$; but should be just $\frac{645}{43}$=15;
Therefore, 645—609=36=first error.

Again. Let us suppose 6 and 7 for their ages 20 years ago; for 6 : 7 in the given ratio at that time, and 6+40 : 7+40 :: 14 : $\frac{658}{46}$; but should be $\frac{690}{46}$=15; therefore, 690—658=32 the second error.

Lastly. $\overline{36-32} : 1 :: \overline{36\times6}-\overline{32\times3} : 30$= age of youngest at first, and 35=age of second; for, 6 : 7 :: 30 : 35, and 30+20=50=age of younger at the time they conversed, and 55 that of the elder; for, $\overline{30+40} : \overline{35+40} :: 14 : 15$ at 20 years afterwards.

Any question we choose, may be resolved by the following rule, deduced from the principles of Double Position, viz.

First, proceed as in Double Position, only let your supposed numbers be as near the truth as possible; and then by analogy, say,

> As the difference of the errors, if alike, or their sum, if unlike,
> Is to the difference of the positions;
> So is the least error,
> To the difference between its position and the first approximate value of the number sought.

Repeat the operation with the approximate value, and the position which gave the least error for a farther approximation; and so on, to any degree of accuracy we may desire.

Let us take for example a problem, involving an adfected quadratic equation, viz.

7. Suppose it was required to find two such numbers, that whether they be added, or multiplied, their sum and product shall each be equal to 5.

First. Suppose 1.36 the lesser, and of course the greater $=3.64$;

Then $1.36+3.64=5$

$1.36\times3.64=4.9504$, but should have been exactly 5; therefore, $5-4.9504=.0496=$ first error.

Secondly. Let us suppose 1.38 to be the lesser number, and of course $3.62=$ greater;

Then $1.38+3.62=5$

$1.38\times3.62=4.9956$, but should be exactly 5; therefore, $5-4.9956=.0044=$ second error.

Now per rule,

$.0496-.0044 : \overline{1.38-1.36} :: 1.38 : .0061$, and $1.38+.0061=1.3861$, the first approximate value for the smaller number, which is very near the truth; for $1.3861+3.6139=5$

and $1.3861\times3.6139=5.00922679$, not quite $\frac{1}{100}$ of a unit too much.

By repeating the operation with this approximate value, we still come nearer. The true numbers are

$$2.5+\sqrt{1.25}, \text{ and } 2.5-\sqrt{1.25};$$

because $2.5+\sqrt{1.52}+2.5-\sqrt{1.25}=5$, and

$$2.5+\sqrt{1.25}\times 2.5-\sqrt{1.25}=6.25\begin{pmatrix}+2.5\sqrt{1.25}\\-2.5\sqrt{1.25}\end{pmatrix}$$

$-1.25=5$.

NOTE 1. These values are easily found by algebra.

8. A gentleman courted a young lady, and as their birth-days happened together, they agreed to make that their wedding-day. On the day of marriage, it happened that the gentleman's age was just double to that of the lady's; that is, as 2 to 1. After they had lived together 30 years, the gentleman observed, that his lady's age drew nearer to his, and that his was only to that of hers, as 2 to $1\frac{3}{7}$; 30 years after this, the same gentleman found his and his lady's ages to be as near as 2 to $1\frac{3}{5}$; at which time they both died. I demand their several ages at the day of their marriage, and of their death; also the reason why the lady's age, which was continually gaining upon her husband's, should, notwithstanding, be never able to overtake it.

Ans. 40 and 20 years were their
respective ages at marriage, and
100, and 80 at their death.

Suppose their ages at marriage to be 30 and 60; then,

$\left\{\begin{matrix}30+30=60\\60+30=90\end{matrix}\right\}$ and $2 : \frac{10}{7} :: 90 : 64\frac{2}{7}$, $64\frac{2}{7}-60 = 4\frac{2}{7}=\frac{30}{7}=$ first error +

Again. Suppose 30 and 35 for their ages; then,

$\left\{\begin{matrix}35+30=65\\70+30=100\end{matrix}\right\}$ and $2 : \frac{10}{7} :: 100 : 71\frac{3}{7}$, $71\frac{3}{7}-65=6\frac{3}{7}=\frac{45}{7}$, second error +

Therefore, per rule, $\frac{45}{7}-\frac{30}{7} : 35-30 :: 30 : 10$, and $30-10=20$, age of the lady at marriage.

40, that of the husband.

And $20+30+30=80$, her age at death.

$40+30+30=100$, his age at death.

The reason why the lady's age drew nearer to the gentleman's, is, that unequal numbers continually augmented by the addition of one common number, or quantity, constantly approach the ratio of equality, as appears plainly by the following example, viz.

Let the numbers 2 and 1 be taken; 2 : 1 is in the ratio of 2, or the ratio of 2 : 1 is 2, for $\frac{2}{1}=2$. Now, if 1 be added to each, they will have a less ratio, viz. 1½, for 3 : 2 $=\frac{3}{2}=1\frac{1}{2}$; if 1 be added again, then 4 : 3 is still a less ratio, for 4 : 3 $=\frac{4}{3}=1\frac{1}{3}$, and *so on*. These numbers being infinitely augmented, by constantly adding to each an unit, the numbers thus augmented will become infinitely near the ratio of equality. Therefore, the reason why the lady's age approached that of her husband's, will appear by comparing the ratio of their ages at marriage, and at death. First, 40 : 20 :: 2 : 1; that is, $\frac{40}{20}=\frac{2}{1}=2$ = ratio of their ages at marriage; and 100 : 80 :: 2 : $1\frac{3}{5}$; but $\frac{100}{80}=1\frac{1}{4}=\frac{2}{1\frac{3}{5}}=\frac{10}{8}$ a less ratio than the former; therefore the reason is evident. QED.

NOTE. I have been more attentive to the rule of Double Position than most others; because I deem it amongst the highest rules of arithmetic.

PERMUTATIONS & COMBINATIONS.

The Permutation of quantities is, the shewing how many different ways any given number of things may be changed, or assume different positions.

The Combination of quantities is, the shewing how often a less number of things can be taken out of a greater, and combined together, without considering their places, or the order they stand in.

PROBLEM 1.

To find the number of permutations, or changes, that can be made of any given number of things, all different from each other,

RULE.

Multiply all the terms of the natural series of numbers, from 1 up to the given number, continually together, and the last product will be the answer required.

EXAMPLES.

1 How many changes of position can a company of 6 persons assume?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 =$

2. How many different numbers can be made of the number 123456789?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 =$

PROBLEM 2.

Any number of different things being given, to find how many changes can be made out of them, by taking any given number of quantities at a time,

RULE.

Take a series of numbers, beginning at the number of things given, and decreasing by 1, to the number of quantities to be taken at a time; the product of all the terms will be the answer required.

EXAMPLE.

How many changes may be rung with 4 bells out of 8? $8 \times 7 \times 6 \times 5 = 1680$. Ans.

PROBLEM 3.

Any number of things being given, whereof there are several things of one sort, several of another, &c.,

to find how many changes may be made out of them all,

RULE.

1. Take the series $1\times2\times3\times4$, &c., up to the number of things given, and find the product of all the terms.

2. Take the series $1\times2\times3\times4$, &c., up to the number of the given things of the first sort, and the series $1\times2\times3\times4$, &c., up to the number of the given things of the second sort, &c.

3. Divide the product of all the terms of the first series, by the joint product of all the terms of the remaining ones, and the quotient will be the answer required.

EXAMPLES.

1. How many variations may be made of the letters in the word *Mississippi?*

$$1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11=39916800.$$

$1=$number of *m*'s $=1$

$1\times2\times3\times4=$number of *i*'s $=24$

$1\times2\times3\times4=$number of *s*'s $=24$

$1\times2=$number of *p*'s $=2$

$1\times24\times24\times2=1152)39916800(34650.$ Ans.

2. How many different numbers can be made of the following figures, 1223334444?

Ans. 12600.

PROBLEM 4.

To find the number of combinations of any given number of things, all different from one another, taking any given number at a time,

RULE.

1. Take the series, 1, 2, 3, 4, &c., up to the number to be taken at a time, and find the product of all the terms.

2. Take a series of as many terms, decreasing by 1, from the given number, out of which the election is to be made, and find the product of all the terms.

3. Divide the last product by the former, and the quotient will be the number sought.

EXAMPLES.

1. How many combinations may be made of six letters out of 12?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 =$ (the number of terms taken at a time)720.

$12 \times 11 \times 10 \times 9 \times 8 \times 7 =$ (a like number of terms, from 12 down)665280, and $665280 \div 720 = 924$. Ans.

2. How many sections of 4, could be made out of a company of 14 men? Ans. 1820.

PROBLEM 5.

To find the number of combinations of any given number of things, by taking any given number at a time; of which there are several things of one sort, and several things of another, &c.

RULE.

Find the number of different forms, which the things, to be taken at a time, will admit of, in the following manner:

1. Place the things so, that the greatest indices may be first, and the rest in order.

2. Begin with the first letter, and join it to the second, third, fourth, &c. to the last.

3. Join the second letter to the third, fourth, &c. to the last; and so on, till they are all done, always rejecting such combinations as have occurred before; and this will give the combinations of all the twos.

4. Join the first letter to every one of the twos; then join the second, third, &c., as before; and it will give the combinations of all the threes.

5. Proceed in the same manner to get the combinations of all the fours, fives, &c., and you will at last get all the several forms of combination, and the number in each form.

6. Having found the number of combinations in each form, add them all together, and the sum will be the number required.

EXAMPLE.

Let the things proposed be $z z z y y x$; *it is required* to find the number of combinations of every 2, of every 3, and of every 4, of these quantities.

Combinations.	Eorms.	Combinations in each.	
$z z, z z, z y, z y, z x$	z^2, y^2	2	
$z z, z y, z y, z x$	$z y, z x, y x$	3	
$z y, z y, z x$		—	
$y y, y x$		5=sum of the 2s.	
$y x$			
	$z^3 =$		1
$z z z, z z y, z z y, z z x$	$z^2 y, z^2 x, y^2 z, y^2 x$		4
$z z y, z z y, z z x$	$z y x$		1
$z y y, z y x$			—
$y y x$			6=sum
$z z z y, z z z y, z z z x$	$z^3 y, z^3 x$	2	of the 3s.
$z z y y, z z y x$	$z^2 y^2$	1	
$z y y x$	$z^2 y x, y^2 z x$	2	
		—	
		5=sum of the 4s.	

Ans. 5 combinations of every 2; 6, of every 3; and 5, of every 4 quantities.

PROBLEM 6.

To find the changes of any given number of things, taken a given number at a time; of which there are several given things of one sort, and several of another, &c.

RULE.

1. Find all the different forms of combination of all the given things, taken as many at a time, as in the question, by problem 5.

2. Find the number of changes in any form, (by problem 3.) and multiply it by the number of combinations in that form.

3. Do the same for every distinct form, and the sum of all the products will give the whole number of changes required.

EXAMPLE.

How many changes can be made of every 4 letters out of these 6; *a a a b b c*? Changes.

$$\left.\begin{array}{rr} a^3b,\ a^3c & 2 \\ a^2b^2 & 1 \\ a^2bc,\ b^2ac & 2 \end{array}\right\} \left.\begin{array}{l} \left\{\begin{array}{l} 1\times2\times3\times4=24 \\ 1\times2\times3\quad\ \ =6 \end{array}\right. \frac{24}{6}=4 \\ \left\{\begin{array}{l} 1\times2\times3\times4=24 \\ 1\times2\times2\quad\ \ =4 \end{array}\right. \frac{24}{4}=6 \\ \begin{array}{l} 1\times2\times3\times4=24 \\ 1\times2 \end{array} \frac{24}{2}=12 \end{array}\right\}$$

$$\text{Therefore, } \begin{array}{l} 2\times4=8 \\ 1\times6=6 \\ 2\times12=24 \\ \hline \end{array}$$

38 number of chnages required.

PROBLEM 7.

To find the compositions of any number, in an equal number of sets the things being all different,

RULE.

Multiply 9 continually, as often as the number of companies.

EXAMPLE.

Suppose there are 5 companies, of 12 men each; it is required to find how many ways 9 men may be chosen, one out of each company.

$9\times9\times9\times9\times9=59049$. Ans.

Note. Many more curious problems might be raised on the rules of Permutations and Combinations just discussed; but the subject being more curious than useful, let what has been said suffice.

ARITHMETICAL PROGRESSION.

When several quantities increase or decrease by the same common difference, they form an Arithmetical series. Such, for example, are these:

$1, 2, 3, 4, 5,$ &c.
Or, $8, 6, 4, 2,$ &c.
Or, $a, a+d, a+2d,$ &c.
Or, $z, z-d, z-2d,$ &c.

In an Arithmetical series, let

a denote the first term.
d the common difference.
n the number of terms.
z the last term.
And s the sum of all the terms.

Then the series will stand thus:

$$a,\ a+d,\ a+2d,\ a+3d,\ a+4d \dots z.$$

Here it is manifest, by simply inspecting the series, that its nth term will be $a+(n-1)d$, and, therefore, that $z=a+(n-1)d$.

Again, to find the sum of all the terms, let the series be written twice, in an inverse order, so that the first term may be over the last, and the second over the last but one, &c., thus:

$$\begin{array}{lllll} & a, & a+d, & a+2d, & a+3d \dots z \\ & z, & z-d, & z-2d, & z-3d \dots a \\ \text{Or,} & 1, & 1+2, & 1+4, & 1+6 \ \dots 7 \\ & 7, & 7-2, & 7-4, & 7-6 \ \dots 1 \end{array}$$

Then it is plain, that the sum of every two terms in the same perpendicular rank, is equal to the sum of the two extreme terms of the series; therefore, the sum of the sums of all these pairs of terms, is equal to the sum of the extreme, taken as often as there are terms. But if we take the sums of all these pairs of terms, and add them together, the result will evidently be twice the sum of all the terms of the series; therefore, twice the sum of all the terms of the series, is equal to the sum of the extremes taken as often as there are terms.

That is, $2s=n(a+z)$; therefore, $s=\frac{n(a+z)}{2}$.

Or, $2\times16=4(1+7)$; therefore, $16=\frac{4(1+7)}{2}$.

The two equations,

$$z=a+(n-1)d,\ s=\frac{n(a+z)}{2}$$

$$\text{Or, } 7=1+(4-1)2,\ 16=\frac{4(1+7)}{2}$$

express the relations which the five quantities, a, d, n, z, s, have to one another; or that 1, 2, 4, 7, 16, have to each other; (but here we use the numbers for the first term, common difference, &c., they must vary as the case may require,) and by means of them, when any three of the five quantities are given, the remaining two or either of them, may be found, as in the following examples.

EXAMPLE 1.

The first term of an arithmetical series is 2, the common difference 3, and the number of terms 50; required, the last term and sum.

Here we have given $a=2$, $d=3$, $n=50$, to find z and s. Let these values of a, d, and n, be substituted in the first equation, $z=a+(n-1)d$, and we get $z=2+\overline{49}\times3=149$. Again, let the values of n, a, and that just now found for z, be substituted in the second equation $s=\frac{n(a+z)}{2}$, and we get $s=\frac{50\times151}{2}$ $=3775$.

EXAMPLE 2.

The first term of an arithmetical series is 3, the common difference 2, and the sum 120; required, the number of terms.

Here we have $a=3$, $d=2$, and $s=120$, to find n. Let these numbers be substituted in the two equations, instead of the letters which denote them, and we get $z=1+2n$, $240=3n+nz$.

By means of these equations, let z be exterminated, and the result is the quadratic equation $n^2+2n=120$; hence, n is found to be $+10$, or -12; but as it is manifestly only the positive root that can be admitted, we have $n=10$.

From the preceding principles we gather, that of 5 things, 10 combinations of 3 of the 5 may be made, for, $\frac{5\times4\times3}{1\times2\times3}=10$. Hence arises four ways of finding each of the five constituent parts of an arithmetical series; and for which purpose, it is necessary to employ exactly twenty problems. But first, we will shew the theorems, or final equations, in algebraic form; which are, in fact the rules for computing the several cases, and may be readily written out in words.

Note. On the two following pages, are to be found twenty formulas, including all the cases of Arithmetical Progression. Sets of formulas are to be found in other books; but, I confess, I never found them without many blunders. I trust that mine are correct and general.

From the preceding principles, and primitive equations arising from them, and by the resolution or variation of the said primitive equations, the following formulas arise; which shew the method of performing all possible cases in Arithmetical Progression.

Case.	Given.	Required.	Solution or final equations.
1.	$a, z, n,$	d	$=\frac{z-a}{n-1}$
		s	$=\frac{\overline{a+z}\times n}{2}$

Case.	Given.	Required.	Solution or final equations.
2.	$a, z, d,$	n	$= \frac{z-a}{d}+1$
		s	$= \frac{\overline{z+a}\times\overline{z-a+d}}{2d}$
3.	$a, z, s,$	d	$= \frac{\overline{z+a}+\overline{z-a}}{2s-z-a}$
		n	$= \frac{2s}{a+z}$
4.	$a, d, s,$	n	$= \frac{\sqrt{(2a-d)^2+8ds}-\overline{2a-d}}{2d}$
		z	$= \frac{\sqrt{(2a-d)^2+8ds}-d}{2}$
5.	$a, d, n,$	z	$= \overline{n-1}\times d+a$
		s	$= n\times\overline{a+n-1\times\frac{d}{2}}$

In order to shew how to apply the above formulas, let us suppose the following arithmetical series given, viz. 2, 5, 8, 11, consisting of 4 terms;

Then, $a = 2$ = first term.
$d = 3$ = common difference.
$n = 4$ = number of terms.
$z = 11$ = last term.
$s = 26$ = sum of series.

Now, let these values of a, d, n, z, and s be substituted in the equations above, &c., and every thing may be found in round numbers. Take the first equation:

Say, given 2, 11, 4; then $\frac{11-2}{4-1}=3=d$, and $\frac{\overline{2+11}\times 4}{2}=26=s$, And so of all the rest.

Case.	Given.	Required.	Solution or final equations.
6.	$a, n, s,$	d	$= \dfrac{2\times\overline{s-an}}{\overline{n-1}\times n}$
		z	$= \dfrac{2s}{n}-a$
7.	$z, d, s,$	a	$= \dfrac{d\pm\sqrt{(2z+d)^2-8ds}}{2}$
		n	$= \dfrac{2z+d\pm\sqrt{(2z+d)^2-8ds}}{2d}$
8.	$z, n, s,$	a	$= \dfrac{2s}{n}-z$
		d	$= \dfrac{2\times\overline{nz-s}}{\overline{n-1}\times n}$
9.	$z, n, d,$	a	$= z-\overline{n-1}\times d$
		s	$= n\times\overline{z-\overline{n-1}\times\dfrac{d}{2}}$
10.	$d, n, s,$	a	$= \dfrac{s}{n}-\dfrac{d\times\overline{n-1}}{2}$
		z	$= \dfrac{s}{n}+\dfrac{d\times\overline{n-1}}{2}$

The following are forms of arithmetical serieses, which often occur in mathematical dissertations:

1, 3, 5, 9, &c.*
12, 9, 6, 3, 0—3—6—9—12.
1, 1½, 2, 2½, 3, 3½, &c.
1, 5, 9, 13, 17, 21, &c.
3, 6, 9, 12, 15, 18.
18, 15, 12, 9, 6, 3, &c.

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* This particular series, has a peculiar property, viz. that the sum of any number of terms (x) is equal to the square (x^2) of that number. See foot of next page.

Here follows the problems in order, according to the preceding formulas.

PROBLEM 1.

The first term, the last term, and the number of terms, being given, to find the common difference.

The extremes are 3 and 39, and the number of terms 19; what is the common difference?

By formula 1, per table, $\frac{39-3}{19-1}=2$. Ans.

PROBLEM 2.

The first term, the last term, and the number of terms, being given, to find the sum of all the terms.

The extremes of an arithmetical series are 3 and 39, and the number of terms 19; required, the sum of the series.

By formula 2, $\frac{\overline{3+39}\times 19}{2}=399$. Ans.

PROBLEM 3.

Given, the extremes, and the common difference, to find the number of terms.

The extremes are 3 and 39, and the common difference 2; what is the number of terms?

By formula 3, $\frac{39-3}{2}+1=19$. Ans.

PROBLEM 4.

The extremes and common difference given, to find the sum of the series.

If the extremes are 3 and 39, and the common difference 2; what is the sum of the series?

By formula 4, $\frac{\overline{3+39}\times\overline{39-3+2}}{2\times 2}=399$. Ans.

For, $0+1=$sum of 1 term $=1^2$ or 1.
$1+3=$sum of 2 terms$=2^2$ or 4.
$4+5=$sum of 3 terms$=3^2$ or 9.
$9+7=$sum of 4 terms$=4^2$ or 16.
$16+9=$sum of 5 terms$=5^2$ or 25, &c.

PROBLEM 5.

The extremes and the sum of the series given, to find the common difference.

Let the extremes be 3 and 39, and the sum of the series be 399; what is the common difference?

By formula 5. $\frac{\overline{39+3}\times\overline{39-3}}{399\times2-39-3}=2.$ Ans.

PROBLEM 6.

The extremes and sum of the series given, to find the number of terms.

Let the extremes be 3 and 39, and the sum of the series 399; what is the number of terms?

By formula 6. $\frac{2\times399}{3+39}=19.$ Ans.

PROBLEM 7.

The first term, the common difference, and the sum of the series given, to find the number of terms.

If the first term be 3, the common difference 2, and the sum of the series 399; required, the number of terms.

By formula 7.

$$\frac{\sqrt{(\overline{2\times3}-2)^2+8\times2\times399}-\overline{2\times3-2}}{2\times2}=19.\ \text{Ans.}$$

PROBLEM 8.

The first term, the common difference, and the sum of the series given, to find the last term.

If the first term be 3, the common difference 2, and the sum of the series 399; what is the last term?

By formula 8.

$$\frac{\sqrt{(\overline{2\times3}-2)^2+8\times2\times399}-2}{2}=39.\ \text{Ans.}$$

PROBLEM 9.

The first term, the common difference, and the number of terms given, to find the last term.

If the first term be 3, the common difference 2, and the number of terms 19; what is the last term?

By formula 9. $\overline{19-1}\times 2+3=39$. Ans.

PROBLEM 10.

The first term, common difference, and number of terms given, to find the sum of the series.

If the first term is 3, the common difference 2, and number of terms 19; what is the sum of the series?

By formula 10. $19\times\overline{3+\overline{19-1}\times\frac{2}{2}}=399$. Ans.

PROBLEM 11.

Given, the first term, the number of terms, and the sum of the series, to find the common difference.

If the first term be 3, the number of terms 19, and the sum of the series 399; what is the common difference?

By formula 11. $\frac{2\times\overline{399-3\times 19}}{19-1\times 19}=2$. Ans.

PROBLEM 12.

Given, the first term, number of terms, and the sum of the series, to find the last term.

First term 3, number of terms 19, and sum 399.

By formula 12. $\frac{2\times 399}{19}-3=39$. Ans.

PROBLEM 13.

The common difference, last term, and sum of the progression given, to find the first term.

If the common difference be 2, the last term 39, and the sum of the terms 399; required, the first term. By formula 13.

$$\frac{\sqrt{(2\times 39+2)^2-\overline{8\times 2\times 399}}\pm 2}{2}=3.\ \text{Ans.}$$

PROBLEM 14.

Given, the common difference, the last term, and sum of the series, to find the number of terms.

If the common difference be 2, the last term 39, and the sum of the terms 399; I demand the number of terms. By formula 14.

$$\frac{2\times39+2\mp\sqrt{(2\times39+2)^2-8\times2\times399}}{2\times2}=19.$$ An.

PROBLEM 15.

Given, the last term, the number of terms, and the sum of the series, to find the first term.

If the last term be 39, the number of terms 19, and the sum of the series 399; what is the first term?

By formula 15. $\frac{2\times399}{19}-39=3.$ Ans.

PROBLEM 16.

Given, the last term, the number of terms, and the sum of the terms, to find the common difference.

If the last term be 39, the number of terms 19, and the sum of the series 399; required, the common difference.

By formula 16. $\frac{2\times\overline{19\times39-399}}{\overline{19-1}\times19}=2.$ An.

PROBLEM 17.

The common difference, number of terms, and the last term given, to find the first term.

If the common difference be 2, the number of terms 19, and the last term 39; required, the first term.

By formula 17. $39-\overline{19-1}\times2=3.$ Ans.

PROBLEM 18.

The common difference, the number of terms, and last term given, to find the sum of the series.

If the common difference be 2, number of terms 19, and the last term 39; what is the sum of the series?

By formula 18. $19\times\overline{39-\overline{19-1}\times\tfrac{2}{2}}=399.$ Ans.

PROBLEM 19.

The sum of the series, the number of terms, and the common difference given, to find the first term.

If the sum of the series be 399, the number of terms 19, and the common difference 2; what is the first term?

By formula 19. $\frac{399}{19}-\frac{2\times\overline{19-1}}{2}=3$. Ans.

PROBLEM 20.

The sum of the terms, number of terms, and the common difference given, to find the last term.

If the sum of the series be 399, the number of terms 19, and the common difference 2; what is the last term?

By formula 20. $\frac{399}{19}+\frac{2\times\overline{19-1}}{2}=39$. Ans.

Sundry examples follow, to exercise the foregoing rules. If any three of the five parts of an arithmetical series be given, the remaining ones may be had by some one other of the rules heretofore exemplified.

1. A goldsmith had an elegant gold watch-chain, which had just 40 links in it; a young man looking at it, said he should like to purchase it, if the price was any where in reason. The goldsmith replied, that if he would give him 1 cent for the first link in the chain, and 4 for the second, increasing the price of each succeeding link 3 cents, he should have it. How much was the price? Ans. $23.80.

2. A smart young fellow said he could gather up 100 apples (placed one yard asunder) into a basket placed beside the first apple, in 20 minutes; but if one yard behind it, he thought it would tough him. Another said he would bet a dozen of wine, that if

the basket be placed three yards behind the first apple, that he could not gather them singly into it in an hour. What distance would he have to travel (starting at the basket) to take them up?

Ans. 5 miles, 1700 yards.

3. If the first term of an arithmetical series be 1, the last term 2, and number of terms 100; required, the common difference and sum of the series.

Ans. Sum 150, common difference $\frac{1}{99}$.

4. The sum of the series is 1000, number of terms 1000, and last term 100; required, the first term, and common difference.

Ans. —98, common difference $\frac{22}{111}$.

5. What is the difference between the sum of the first 1000 natural numbers, taken in order from unity, and the sums of the first 100, first and second hundreds, second and third hundreds, &c, to the ninth and tenth hundreds added together?

Ans. 450.

6. Required, the sum of 1000 terms of the series of odd numbers, 1, 3, 5, 7, 9, &c.

Ans. 1000000.

7. What would the first term of a decreasing series and its sum be, when the number of terms and common difference are 1000 and 3, and last term 1?

Ans. 2998 first term, and 1499500 sum.

8. Given, the sum of the series $=$ 8000, number of terms $=$ 4000, and common difference 2, to find the last term. Ans. 4001.

GEOMETRICAL PROGRESSION.

When a number of quantities increase by the same multiplier, or decrease by the same divisor, they form a Geometrical series.

This common multiplier, or divisor, is called the *common ratio*.

Thus, $1, 2, 4, 8, 16$, &c.

Or, $1, \frac{1}{10}, \frac{1}{100}, \frac{1}{1000}$, &c.

Or, in general, a, ar, ar^2, ar^3, &c. is a geometrical series. And here r may be either a whole number, or a fraction. If r be greater than 1, the series will increase; but if it be less than 1, the series will decrease.

Let a denote the first term.
r the common multiplier, or ratio.
n the number of terms.
z the least term.
s the sum of all the terms.

Then the series will be $a, ar, ar^2, ar^3, ar^4 \ldots . z$.

And since it appears that the exponent of r, in any term, is one less than the number expressing the place of that term, it is evident, that $z=ar^{n-1}$.

Now, $s=a+ar+ar^2+ar^3 \ldots . +ar^{n-1}$.

Therefore, multiplying both sides of the equation by r, we have $rs=ar+ar^2+ar^3 \ldots ar^{n-1}+ar^n$.

Let the first of these equations be subtracted from the second, and we have $rs-s=-a+ar^n$, or $(r-1)s=n(r^n-1)$.

Therefore, $s=\dfrac{a(r^n-1)}{r-1}$

Hence it appears, that the relations of the five quantities, a, r, n, z, s, to one another, are expressed by the two equations $z=a(r^{n-1})$. $s=\dfrac{a(r^n-1)}{r-1}$

And by these, having given any three of the four quantities, a, r, n, s. the remaining quantity may be found by the resolution of equations. If, however, n be not a small number, the cases of this problem will be most conveniently resolved by logarithms.

If we suppose the series to decrease, (in which case r will be a fraction,) and also that the number of terms is indefinitely great, then the formula for finding the sum, in this case, may be investigated as follows:

From the first equation we have $r^n=\frac{rz}{a}$, and therefore, $r^n-1=\frac{rz}{a}-1$, and $a(r^n-1)=rz-a$. Therefore, from the 2d equation, we find $s=\frac{rz-a}{r-1}$, or rather, since $r \angle 1$, $s=\frac{a-rz}{1-r}$; but when the number of terms is indefinitely great, and the series decreases, z is less than any assignable quantity, and is therefore to be reckoned as 0, or $=0$; therefore, in this case, $rz=0$, and $s=\frac{a}{1-r}$

EXAMPLE.

It is required to find the sum of the infinite series 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c. to infinity.

Here $a=1$, $r=\frac{1}{2}$;

Therefore, $s=\frac{1}{1-\frac{1}{2}}=\frac{2}{2-1}=2$: Ans.

From the preceding doctrine we gather, that the constituent parts of a geometrical series are five, as well as in Arithmetical Progression; and that of the five, any three being given, the rest are had by an easy process, as will be shewn by formulas adapted to every case. We observed before, that of five things, ten combinations of every three of them may be made, which will all be different; hence also arises twenty problems, which shew four different ways of finding each of the five parts. The formulas arise principally out of these two, viz. $z=(r^{n-1})\times a$ and $s=\frac{a(r^n-1)}{r-1}$. By what is called the resolution of equations.

In order to illustrate numerically that the above formulas, or equations, are true, let us suppose the series 2, 6, 18, given,

Then the first term a will be $=2$
r 3
n 3
z 18
s 26

By substituting in the first of the above equations, the numbers answering to the letters, then $18=2(3^{3-1})$, and $26=\frac{2(3^3-1)}{3-1}$, and so of all the rest.

Note. I have observed (as I before noticed, in Arithmetical Progression) that in several books, sets of formulas are to be found for each case; but I have not seen one perfect set. I confidently trust, that I have fully remedied the evil, by introducing correct and general formulas for each case.

TABLE OF FORMULAS, &c.

Case.	Given.	Required.	Solution or final equations.
1.	$a, r, n,$	$z =$	ar^{n-1}
		$s =$	$\frac{a(r^n-1)}{r-1}$

Case.	Given.	Required.	Solution or final equations.
2.	$a, r, z,$	s	$= z+\frac{z-a}{r-1}$
		n	$= \frac{{}^{*}\text{Log. } z-\text{Log. } a}{\text{Log. } r}+1$
3.	$a, r, s,$	z	$= \frac{s\times\overline{r-1}+a}{r}$
		n	$= \frac{\text{Log. }\overline{rs+ra-s-a}-\text{log. } a}{\text{Log. } r}$
4.	$a, z, s,$	r	$= \frac{s-a}{s-z}$
		n	$= \frac{\text{Log. } z-\text{log. } a}{\text{Log. }\overline{s-a}-\text{log. }\overline{s-z}}+1$
5.	$a, n, s,$	r	$= \sqrt{\frac{s}{a}-\frac{3}{4}}-\frac{1}{2}$
		z	$= a\times(\sqrt{\frac{s}{a}-\frac{3}{4}}-\frac{1}{2})^2$
6.	$a, n, z,$	r	$= \left(\frac{z}{a}\right)^{\frac{1}{n-1}}$
		s	$= \frac{z-a}{\left(\frac{z}{a}\right)^{\frac{1}{n-1}}-1}+z$

---

* Here Log. $z=18=1.255272$
Log. $a=\ 2=\ .301030$

Log. $r^{n-1}=\ 9=\ .954242$
Log. $r=\ 3=\ .477121$

Log. $n=\ 3=\ .477121$ Ans.

Or, $\frac{1.255272-.301030}{.477121}=2$, and $2+1=3=n$. Ans.

Case.	Given.	Required.	Solution or final equations.
7.	$r, n, z,$	a	$= \frac{z}{r^{n-1}}$
		s	$= \frac{z + z - \frac{z}{r^{n-1}}}{r-1}$
8.	$r, n, s,$	a	$= \frac{r-1}{r^n-1} \times s$
		z	$= \frac{r^n - r^{n-1}}{r^n-1} \times s$
9.	$r, z, s,$	a	$= s - \overline{r \times \overline{s-z}}$
		n	$= \frac{\text{Log.}\, z - \text{L.}\, s - r \times \overline{s-z}}{\text{Log.}\, r} + 1$
10.	$n, z, s,$	a	$= \frac{2s - z - \sqrt{4sz - 3z^2}}{2}$ If this be put $= d$, then
		r	$= \sqrt{\frac{s}{d}} - \frac{3}{4} - \frac{1}{2}$

APPLICATION OF THE PRECEDING FORMULAS.

PROBLEM 1.

Given, one of the extremes, the ratio, and the number of the terms of a geometrical series, to find the other extreme.

If the first term be 2, the ratio 3, and the number of terms 3; what is the last term?

By formula 1. $3^{3-1} \times 2 = 18$. Ans.

But if, according to the problem, we did not know which extreme was the greater, it would be known by comparison, after the operation is performed.

PROBLEM 2.

Given, the first term, the ratio, and number of terms, to find the sum of the series.

If the first term be 2, the ratio 3, and the number of terms 3; what is the sum of the series?

By formula 2. $\frac{2(3^3-1}{3-1}=\frac{52}{2}=26$. Ans.

PROBLEM 3.

The first term, the last term, and the ratio, given, to find the sum of the series.

If the first term be 2, the last term 18, and the ratio 3; what is the sum of the series?

By formula 3. $18+\frac{18-2}{3-1}=26$. Ans.

PROBLEM 4.

Given, the extremes and ratio, to find the number of terms.

If the extremes be 2 and 18, and the ratio 3; what is the number of terms?

By formula 4. $\frac{\text{Log. }18-\text{log. }2}{\text{log. }3}+1=3$. Ans.

Or, Log. of 18 = 1.255272
Minus log. of 2 = .301030

Divided by log. of 3 = $\frac{.954242}{.477121}=2$, & $2+1=3$ An.

PROBLEM 5.

Given, the least term, the ratio, and sum of the series, to find the last term.

If the least or first term be 2, the ratio 3, and the sum of the series 26; what is the last term?

By formula 5. $\frac{3^{3-1}\times 26(3-1)}{3^3-1}=18$. Ans.

PROBLEM 6.

Given, the least term, the ratio, and sum of the series, to find the number of terms.

If the least term be 2, the ratio 3, and the sum of the series 26; what is the number of terms?

By formula 6. $\frac{\text{Log. }\overline{26(3-1)+2}-\text{log. }2}{\text{Log. }3}=3$ Ans.

That is, the log. of $54 = 1.732393$
Minus the log of $2 = .301030$

Equal the log. of $3^3 = 1.431363$
This log. ÷ by log. of $3 = .477121$ $\quad \frac{1.431363}{.477121}=3.$ Ans.

PROBLEM 7.

Given, the extremes, and sum of the series, to find the ratio.

If the extremes of a geometrical series be 2 and 18, and the sum of the series 26; what is the ratio?

By formula 7. $\frac{26-2}{26-18}=3.$ Ans.

PROBLEM 8.

Given, the extremes, and sum of the series, to find the number of terms.

If the extremes be 2 and 18, and the sum of the series 26; what is the number of terms?

By formula 8.

$$\frac{\text{Log. }18-\text{log. }2}{\text{Log. }\overline{26-2}-\text{log. }\overline{26-18}}+1=3. \text{ Ans.}$$

PROBLEM 9.

Given, the first term, the number of terms, and the sum of the series, to find the ratio.

If the first term be 2, the number of terms 3, and the sum of the series 26; what is the ratio?

By formula 9. $\sqrt{\frac{26}{2}-\frac{3}{4}}-\frac{1}{2}=3.$ Ans.

PROBLEM 10.

Given, the first term, the number of terms, and the sum, to find the last term.

If the first term be 2, the number of terms 3, and the sum of the series 26; what is the ratio?

By formula 10. $2\left(\sqrt{\frac{26}{2}-\frac{3}{4}}-\frac{1}{2}\right)^2=18$. Ans.

Or, $2\times\left(\sqrt{12.25}-\frac{1}{2}\right)^2=2\times 3^2=18$.

PROBLEM 11.

Given, the extremes, and number of terms, to find the ratio.

If the extremes be 2 and 18, and the number of terms 3; what is the ratio?

By formula 11. $\overline{\frac{18}{2}}\Big|^{\frac{1}{3-1}}=\sqrt{9}=3$. Ans.

PROBLEM 12.

Given, the first term, the last term, and the number of terms, to find the sum of the series.

If the first and last terms be 2 and 18, and the number of terms 3; required, the sum of the series.

By formula 12. $\frac{18-2}{\overline{\frac{18}{2}}\Big|^{\frac{1}{3-2}}-1}+18=26$. Ans.

Or, $\frac{16}{\sqrt{9}-1}+18=26$ as before.

PROBLEM 13.

Given, the ratio, the number of terms, and the last term, to find the first.

If the ratio be 3, the number of terms 3, and the last term 18; required, the first term.

By formula 13. $\frac{18}{3^{3-1}}=2$. Ans.

PROBLEM 14.

Given, the ratio, the number of terms, and the last term, to find the sum of the series.

If the ratio be 3, the number of terms 3, and the last term 18; required, the sum of the series.

By formula 14.

$$18+\frac{18-\frac{18}{3^{3-1}}}{3-1}=18+\frac{16}{2}=26. \text{ Ans.}$$

PROBLEM 15.

Given, the ratio, the number of terms, and the sum of the series, to find the first term.

If the ratio be 3, the number of terms 3, and the sum of the series 26; required, the first term.

By formula 15. $\frac{3-1}{3^3-1}\times 26=2$. Ans.

PROBLEM 16.

Given, the same as above, to find the last term.

By formula 16. $\frac{3^3-3^{3-1}}{3^3-1}\times 26=18$. Ans.

PROBLEM 17.

Given, the ratio, the last term, and the sum of the series, to find the first term.

If the ratio be 3, the last term 18, and the sum of the series 26; what is the first term?

By formula 17. $26-3\times\overline{26-18}=2$. Ans.

PROBLEM 18.

Given, the same as above, to find the number of terms.

By formula 18.

$$\frac{\text{Log. }18-\text{log. }\overline{26-3\times\overline{26-18}}}{\text{Log. }3}+1=3. \text{ Ans.}$$

PROBLEM 19.*

Given, the number of terms of a geometrical series equal 3, *the last term equal* 18, *and the sum of the series equal* 26, *to find the first term.*

By formula 19.

$$\frac{2\times26-18-\sqrt{4\times26\times18-3\times18\times18}}{2}=2.\text{ Ans.}$$

PROBLEM 20.

Given, the number of terms of a geometrical series equal 3, *the last term equal* 18, *and the sum of the series equal* 26, *to find the ratio.*

By formula 20. $\sqrt{\frac{26}{2}}-\frac{3}{4}-\frac{1}{2}=3$. Ans.

METHOD SECOND.

Divide the sum of the series by the difference between the sum and the last term; the quotient will give the ratio, and the remainder, after the operation, the first term.

EXAMPLE.

If the number of terms be 3, the last term 18, and the sum of the series 26; required, the first term and the ratio.

Operation. 26
18
—
8)26(3 ratio.
24
—
2 first term.

---

* Let it be remarked here, that writers on mathematics, such as Webber, and some others, have been greatly puzzled to give proper and commodious theorems for the solution of problems 19 and 20; which are, in fact, as simple as most of the other cases. I cannot divine the reason of this perplexity, unless it arose from the cut-worm of science, viz. ignorance.

EXAMPLES TO EXERCISE SOME OF THE FOREGOING RULES.

First let us put down a few of such serieses of simple numbers, as generally occur in practical questions, together with their indices.

	0	1	2	3	4	5	6	7	Indices.
1st.	1,	2,	4,	8,	16,	32,	64,	128	Powers.
	1	*a*	*aa*	*aaa*	*aaaa*	*aaaaa*	*aaaaaa*	*aaaaaaa*	
	1	2	3	4	5	6	7	8	
2d.	2,	4,	8,	16,	32,	64,	128,	256	
	0	1	2	3	4	5	6	7	
3d.	3,	6,	12,	24,	48,	96,	192,	384	
	0	1	2	3	4	5	6	7	
4th.	2,	6,	18,	54,	162,	486,	1458,	4374	
	0	1	2	3	4	5	6	7	
5th.	1,	3,	9,	27,	81,	243,	729,	2187	
	1,	a,	a^2,	a^3,	a^4,	a^5,	a^6,	a^7	

If we desired to find any assigned term of series, 1st. as the 8th, we have nothing to do but raise the common multiplier, or ratio, 2, to that power whose indice is one less than the number of terms; thus, $2\times2\times2\times2\times2\times2\times2=2^7=128$. Or, by putting down a few of the first terms, with their indices, thus, 1^0, 2^1, 4^2, 8^3, 16^4, &c. $8^3\times16^4=128$, as before. Now, the sum of the indices being just 7, the product of the two terms is equal to the 7th power of the ratio. The same thing may be observed of series 5th; for, suppose the 8th term of it was required, $3\times3\times3\times3\times3\times3\times3=3^7=2187$; but $27^3\times81^4=3^7=2187$, as before: the product of the quantities, the sum of whose indices is just 1 less than the number of terms to the 8th place.

In serieses such as No. 2d, beginning with 2, 4, &c. the 8th term is found as before, only the 7th power of the ratio must be doubled, quadrupled, &c. according as it begins with 2 or 4; or by multiplying any two terms together, the sum of whose indices is equal to that of the required term.

In serieses such as 3d and 4th, we observe the same rule, viz. raise the ratio to a power whose index is 1 less than the required term, and multiply the ratio raised to such power by the first term, and the thing is done. Suppose we desired to find the 8th terms of the 3d and 4th series, $2\times2\times2\times2\times2\times2\times2\times3=2^7\times3=384$; or $\frac{24^3\times48^4}{3}=384$. Secondly, $3\times3\times3\times3\times3\times3\times3\times2=4374$, or $\frac{54^3\times162^4}{2}=4374$, as before.

From which, it can be easily understood, the method of finding any assigned term of a geometrical series; and from the theorems given in the former part of this article, the sum, or any other part belonging to it, may be easily had.

QUESTIONS TO EXERCISE THE PRECEDING RULES.

1. A person bought 50 yards of broad cloth, and was to give 1 cent for the first yard, 2 for the second, 4 for for the third, and so on, in geometrical proportion; how much will the last yard come to, and what the price of the whole?

0 1 2 4 4 5 6
1, 2, 4, 8, 16, 32, 64, &c.

$64\times64=4096=$ that whose index is 12, and $4096\times4096=16777216=$ that whose index is 24. $16777216\times16777216=281474976710656=$ that whose index is $48=$ the 49th term of the progression; and, $281474976710656\times2=562949953421312=$ the last or 50th term, and \$5629499534213.12 = price of the 50th yard, and $\frac{2^{50}-1}{2-1}\times1=$ 1125899906842621, sum of the whole series, which gives \$11258999068426.21, price of the whole.

2. A man bargained for 26 horses; was to give 2 cents for the first, 6 for the second, 18 for the third,

&c. in triple proportion geometrical; what was the price of the last horse, and of the whole?

$$\begin{array}{rl} 0\quad 1\quad 2\quad 3\quad 4\quad 5 & \\ 2,\ 6,\ 18,\ 54,\ 162,\ 486 & \\ 486 & \\ \hline 2)236196 & \\ \hline 118098 & =\text{index } 10. \\ 118098 & \\ \hline 2)13947137604 & \\ \hline 6973568802 & =20\text{th index.} \\ \times 486 & =5\text{th index.} \\ \hline 2)3389154437772 & \\ \hline 1694577218886 & =\text{index } 25\text{th}= \end{array}$$

26th term of the progression; and $16945772188.86 =price of the 26th horse. Again, $\frac{3^{26}-1}{3-1}\times 2 =$ 508373165657=sum of the reries. Therefore, $5083731656.57=price of the whole drove.

3. Required, the sum of 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, &c. continued to 12 terms. Ans. $\frac{265720}{177147}$.

4. If a man was to engage to pay another, 1 cent for the first month, 10 for the second, 100 for the third, and so on, in a tenfold ratio, for 12 month's service; how much would his wages amount to?

Ans. $\frac{10^{12}-1}{10-1}\times 1=111111111111$ cents, =$1111111111.11, a sum sufficient to pay a thousand times the national debt.

5. A man agreed with his neighbour for a team of 4 horses; and was to give 10 cents for the first horse, 10 times as much for the second, increasing the price of each horse in a tenfold ratio; what was the price of the team? Ans. $1111.10.

6. If a man was to work 20 days for the following wages, viz. at 1 mill for the first day's work, 4 for the second, 16 for the third, and so on, increasing each day's wages in quadruple, or fourfold proportion; required, the amount of his wages.

Ans. 366503875.92½.

HARMONICAL PROPORTION.

Harmonical Proportion is that, which is between those numbers that assign the lengths of musical intervals, or the lengths of strings sounding musical notes; and of three numbers it is, when the first is to the third, as the difference between the first and second, is to the difference between the second and third, as the numbers 3, 4, and 6; for 3 : 6 :: 4—3 : 6—4. If the lengths of strings be as the numbers 3, 4, and 6, they will sound an octave 3 to 6, a fifth 2 to 3, and a fourth 3 to 4.

Again, between four numbers, when the first is to the fourth, as the difference between the first and second, is to the difference between the third and fourth, as in the numbers 5, 6, 8, 10; for 5 : 10 :: 6—5 : 10—8, and strings of the lengths 5, 6, 8, and 10 will sound an octave 5 to 10; a sixth greater, 6 to 10; a third greater, 8 to 10; a third less, 5 to 6; a sixth less, 5 to 8; and a fourth, 6 to 8.

A series of numbers in Harmonical Proportion, is reciprocally, as another series in Arithmetical Proportion.

Harmonical, 10 .. 12 .. 15 .. 20 .. 30 .. 60
Arithmetical, 6 .. 5 .. 4 .. 3 .. 2 .. 1

In these serieses, 10 : 12 :: 5 : 6; and 12 : 15 :: 4 : 5; 15 : 20 :: 3 : 4; and so of the rest. Hence, it is perceivable, that those serieses have an obvious relation to, and dependence on each other.

Let a, b, c, be three numbers in musical proportion: Then by what is shewn above, $a : c :: a-b : b-c$; but when b is greater than a, and c greater than b, then,

$a : c :: b-a : c-b$, by multiplying means and extremes $ac-ab=bc-ac$; or from the first, $ab-ac=ac-bc$ but from the last but one, by the resolution of equations, $2ac-bc=ab$ and $c=\frac{ab}{2a-b}=$1st formula; II. $b=\frac{2ac}{a+c}$; III. $a=\frac{bc}{2c-b}$. Hence, if the numbers 3, 4, and 6 were in question, and any two of them given to find the remaining one, it would only be necessary to substitute in the above formulas the figures answering to the place of the letters, and the thing is done; for in formula 1st, $\frac{3\times4}{2\times3-4}=\frac{12}{2}=6$; 2d, $\frac{2\times3\times6}{3+6}=4$; and 3d, $\frac{4\times6}{2\times6-4}=3$

EXAMPLE.

Suppose it were required to find a musical mean proportional between the monochord $50=c$, and the octave $25=a$; then by theorem 2d, $\frac{2ac}{a+c}=\frac{2\times25\times50}{25+50}=\frac{2500}{75}=\frac{100}{3}=33\frac{1}{3}=b$, the length of that chord called a fifth.

If there be four numbers in musical proportion, as a, b, c, d, then, since it is that $a : d :: a-b : c-b$, we have $ac-ab=ad-db$; but if d be greater than c, and b greater than a, then $a : d :: b-a : d-c$, and by multiplying means and extremes, $ad-ac=db-da$; from which, 1. $\frac{db}{2d-c}=a$; 2. $\frac{a}{d}\times\overline{2d-c}=b$; 3. $\frac{2ad-db}{a}=c$; 4. $\frac{ac}{2a-b}=d$. Now it appears, that any three of the four quantities being given, the remaining one may be found.

Let 10, 8, and 6 be given, to find a fourth harmonical proportion. By substitution of the figures in the first formula, $\frac{10\times6}{2\times10-8}=5$, the octave; for it must be noticed, that the octave is always half the monochord, and so of all the rest. From what has been said, the harmonic divisions of the monochord, to sound the pure concords, will be as follow, viz. in the annexed table.

TABLE.

Names of the chords.	Pure concords.	Equal divisions.	Errors.
Whole string,	10.*	10.‖	
♮ second,		9.439	
♯ second,		8.909	
Lesser third,	8.333⅓	8.409	♮ $\frac{1}{13}$
Greater third,	8.†	7.937	♯ $\frac{1}{13}$
Fourth,	7.5	7.492	♯ $\frac{1}{100}$
♯ fourth,		7.071	
Fifth,	6.666⅔	6.674	♮ $\frac{1}{100}$
Lesser sixth,	6.25	6.3	♮ $\frac{1}{13}$
Greater sixth,	6.‡	5.946	♯ $\frac{1}{13}$
♮ seventh,		5.612	
♯ seventh,		5.297	
Eighth, or octave,	5.§	5.	

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* $\frac{5\times8}{2\times5-6}=10.$

† $\frac{2\times5\times10-10\times6}{5}=8.$

‡ $\frac{5}{10}\times\overline{2\times10-8}=6.$

§ $\frac{10\times6}{2\times10-8}=5.$

‖ The monochord is a vibrating string, say 10 inches long; all the other divisions of the same, as 9.439, 8.909, shew the lengths of other strings, sounding half and whole tones, one about. Such lengths as vibrate together are concords; where strings of different lengths fall in together, or as many of them as happen to vibrate together at once, form an unison. Much more might be said on this head, if necessary; but my object was only to give a *few* hints. Whoever would see the subject discussed at large, let him consult Martin's Philosophy.

DUODECIMALS.

Duodecimals are the parts of a foot expressed by inches, seconds, thirds, fourths, &c. The ratio those parts have to each other, appears from the following statement, viz.

That 12 fourths make 1 third.
144 fourths, or 12 thirds, 1 second.
1728 fourths, or 12 seconds, 1 inch.
20736 fourths, or 12 inches, 1 foot.

It is plain from the above, that the fourths in a third, are to the fourths in a fourth, in the ratio of 12, for $\frac{12}{1}=12$; and the fourths in a second, are to the fourths in a third, also in the ratio of 12; for $\frac{144}{12}=12$; and so of the rest. Hence the ratio is 12; made up of *duo*, 2, and *decem*, 10. Any thing reckoned by 12ths, is duodecimally reckoned; because the ratio is 12.

DENOMINATIONS OF DUODECIMALS.

12 fourths make	1 third.	Fourths marked thus,	''''
12 thirds	1 second.	Thirds,	'''
12 seconds	1 inch.	Seconds,	''
12 inches	1 foot.	Inches,	*I.*
		Feet,	*F.*

EXAMPLES IN ADDITION.

The ratio being 12, the rule is evident; each rank being so many 12th parts of the next higher rank, the sum of each must be divided by 12; in other words, we must carry by 12.

F.	*I.*	''	'''	''''
141	11	10	8	8
17	10	10	10	10
19	9	9	9	9
164	10	8	11	11

F.	*I.*	''	'''	''''
174	11	8	6	3
117	8	6	9	9
17	7	6	4	11
17	11	11	10	9

EXAMPLES IN SUBTRACTION.

F.	I.	″	‴	⁗	F.	I.	″	‴	⁗
174	11	8	8	6	167	10	10	8	2
14	11	9	9	7	104	9	11	7	9

MULTIPLICATION OF DUODECIMALS.

We must observe, in duodecimals, or cross multiplication, that

Feet multiplied by feet, give feet.*
Feet multiplied by inches, give inches.
Feet multiplied by seconds, give seconds.
Inches multiplied by inches, give seconds.
Inches multiplied by seconds, give thirds.
Seconds multiplied by seconds, give fourths, &c.

Multiplication of duodecimals is used in computing glazier's, painter's, carpenter's, and any work where the content of its surface is required. It is also used by accountants, for ascertaining ship's tonnage, content of bales, packages, &c., and whatever else is measured by feet, inches, and parts of an inch; where either superficies or solidity is desired.

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* In order that the truth of the above assertions may appear, to such as use the rules arising out of them, let us take 3 feet and 4 feet, and multiply them together, thus, $\frac{3}{1}F.\times\frac{4}{1}F.=\frac{12}{1}F.=12F.$; that is, feet multiplied by feet, give feet. Again, let 6 inches $=\frac{6}{12}=\frac{1}{2}$ a foot, be multiplied by 3 feet $=\frac{3}{1}$ feet, thus, $\frac{1}{2}$ or $\frac{6}{12}\times\frac{3}{1}=\frac{18}{12}$ feet $=18$ inches $=1\frac{1}{2}$ feet; that is, feet multiplied by inches, give inches. Again, multiply 6 seconds $=\frac{6}{144}$ of a foot, by 3 feet, thus, $\frac{6}{144}\times\frac{3}{1}=\frac{18}{144}$ of a foot $=18$ seconds $=\frac{1}{8}$ of a foot, for $\frac{18}{144}=\frac{1}{8}$; that is feet multiplied by seconds, give seconds. Again, let 6 inches be multiplied by 6 inches, thus, $\frac{6}{12}\times\frac{6}{12}$ feet $=\frac{36}{144}$ of a foot $=36$ seconds $=3$ inches $=\frac{1}{4}$ of a foot; for, $\frac{36}{144}=\frac{1}{4}$. From which the rest is evident.

EXAMPLES.

1. A saw-log cuts 12 boards, 14 feet 6 inches by 1 foot 4½ inches; how many feet of boards are in the whole?

By Practice.

$4I.=\frac{1}{3}$	14	6	
$6''=\frac{1}{8}$	4	10	
		7	3
	19	11	3
			12
	*F.*239	3	0

F.	*I.*	"
14	6	0
1	4	6
9	7	3
4	10	0
14	6	0
19	11	3
		12
*F.*239	3	0 Ans.

2. Multiply 17*F.* 10*I.* 6" by 9*E.* 7*I.* 5".
Ans. 171*F.* 11*I.* 0" 10''' 6''''.

3. Multiply 140*F.* 3*I.* 7" by 7*F.* 3*I.* 11". Ans.

4. Multiply 77*F.* 7*I.* 3" by 16*F.* 9*I.* 10". Ans.

5. How many cubic feet of stone work are contained in 9 walls, each 30 feet 6 inches long, 9 feet 8 inches high, and 2 feet thick?
Ans. 30*F.* 6*I.* × 9*F.* 8*I.* × 2*F.* × 9 = 5307 feet.

6. How many cubic feet are in a cellar 30 feet 3 inches long, 27 feet 6 inches broad, and 8 feet deep?
Ans. 6655 feet.

7. What is the content of a stone 6 feet 7 inches long, 2 feet broad, and 1 foot 9 inches thick?
Ans. 23*F.* 0*I.* 6".

8. What is the superficial content of a stone 4 feet 9 inches long, and 3 feet 9½ inches broad?
Ans. 18*F.* 0*I.* 1" 6'''.

9. In 40 pieces of square timber, each 20 feet long, 1 foot 4 inches broad, and 1 foot 2 inches thick, how many solid feet?
Ans. 1244*F.* 5*I.* 4".

10. A merchant imports from Dublin, 6 bales of the following descriptions, viz.

	F.	*I.*		*F.*	*I.*		*F.*	*I.*	
No. 1.	2	10	long,	2	4	broad, and	1	9	deep.
2.	2	10		2	6		1	3	
3.	3	6		2	2		1	8	
4.	2	10		2	8		1	9	
5.	2	10		2	6		1	9	
6.	2	11		2	8		1	8	

Required, the solid measure of the *whole in feet*; the number of tons, at 40 feet per ton; and the price of tonnage, at 20 dollars per ton.

Ans. 71*F.* 7*I.* = 1*T.* 31*F.* 7*I.* Price $35.79.

TO FIND SHIP'S TONNAGE BY CARPENTER'S MEASURE.

RULE.*

1. For single decked vessels, multiply the length, breadth at the main beam, and depth of the hold together, and divide the product by 95.

2. For double decked vessels, take half the breadth of the main beam for the depth of the hold, and work as above directed.

EXAMPLES.

1. What is the tonnage of a single decked vessel, whose length is 60 feet, breadth 20, and depth 8 feet?

$$\frac{60\times20\times8}{95}=101\tfrac{1}{19}.\ \text{Ans.}$$

2. What is the tonnage of a double decked vessel, whose length is 65 feet, breadth 21 feet 6 inches, and depth 10 feet 9 inches?

$$\frac{65\times21.6\times10.9}{95}=158\tfrac{13}{95}.\ \text{Ans.}$$

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* If the deck be bolted at any heighth above the wale, the custom is to pay the carpenter for one half of the additional height, that is, one half of the difference being added to the former depth, gives the depth to be used in calculating the tonnage.

TO FIND GOVERNMENT TONNAGE.

If the vessel be double decked, take the length thereof from the fore part of the main stem, to the after part of the stern post, above the upper deck; the breadth thereof, at the broadest part above the main wales, half of which breadth shall be accounted the depth of such vessel; and then deduct from the length three-fifths of the breadth, multiply the remainder by the breadth, and the product by the depth, and divide this last product by 95, the quotient whereof shall be deemed the true content of such ship or vessel in tonnage; and if such ship or vessel be single decked, take the length and breadth, as above directed, deduct from the said length, three-fifths of the breadth, and take the depth from the underside of the deck plank, to the ceiling in the hold, then multiply and divide as aforesaid, and the quotient shall be deemed the tonnage.

EXAMPLES.

1. What is the government tonnage of a single decked vessel, whose length is 69 feet 6 inches, breadth 22 feet 6 inches, and depth 8 feet 6 inches?

69 6
Deduct 13 6

6=½ 56 0
22 6 breadth.

112
112
28

6=½ 1260
8 6

10080
630

95)10710(112$\frac{70}{95}$ tons. Ans.

22 6 breadth.
3

5)67 6

13 6

2. Required, the tonnage of a single decked vessel, whose length is 75 feet 6 inches, breadth 24 feet 8 inches, and depth 9 feet 10 inches. Ans.

3. What is the government tonnage of a double decked vessel, of the following dimensions; length 75 feet 6 inches, breadth 23 feet 4 inches, and depth 11 feet 8 inches?

F. I.
75 6
14 for $\frac{3}{5}$ breadth.
I.
4=$\frac{1}{3}$ 61 6
6=$\frac{1}{2}$ 23 4 breadth.

183
122
20 6
11 6

6=$\frac{1}{2}$ 1435 0
11 8

15785 0
2=$\frac{1}{3}$ 717 6
239 2

95)16741 8(176$\frac{21}{95}$ tons. Ans.

4. What is the government tonnage of a double decked vessel, of the following dimensions; length 82 feet 3 inches, breadth 24 feet 3 inches, and depth 12 feet 1½ inches? Ans. 209$\frac{63}{95}$.

5. Required, the government tonnage of a double decked vessel, of the following dimensions; length 100 feet 9 inches, breadth 30 feet 7 inches, and depth 15 feet 11½ inches. Ans.

TABLE OF CORDAGE.

A cordage table, shewing how many fathoms, feet, and inches of a rope, of any size, not more than 14 inches, make a hundred weight; with the use of the table.

Inches.	Fathoms.	Feet.	Inches.	Inches.	Fathoms.	Feet.	Inches.	Inches.	Fathoms.	Feet.	Inches.	Inches.	Fathoms.	Feet.	Inches.
1	486	0	0	4¼	26	5	3	7½	8	4	0	10¾	4	1	8
1¼	333	3	0	4½	24	0	0	7¾	8	3	6	11	4	0	3
1½	216	3	0	4¾	21	3	0	8	7	3	6	11¼	3	5	7
1¾	159	3	0	5	19	3	0	8¼	7	0	8	11½	3	4	1
2	124	3	0	5¼	17	4	0	8½	6	4	3	11¾	3	3	3
2¼	96	2	0	5½	16	1	0	8¾	6	2	1	12	3	2	3
2½	77	3	0	5¾	14	4	6	9	6	0	0	12¼	3	2	1
2¾	65	4	0	6	13	3	0	9¼	5	4	0	12½	3	2	0
3	54	0	0	6¼	12	2	0	9½	5	2	0	12¾	2	7	8
3¼	45	5	2	6½	11	3	0	9¾	5	0	6	13	2	5	3
3½	39	3	0	6¾	10	4	0	10	4	5	0	13¼	2	4	9
3¾	34	3	9	7	9	5	6	10¼	4	4	1	13½	2	4	0
4	30	1	6	7¼	9	1	6	10½	4	2	2	13¾	2	3	6
												14	2	2	1

USE OF THE TABLE.

At the top of the table, marked inches, fathoms, feet, inches, the first column is the thickness of the rope, in inches and quarters, and the other three, fathoms, feet, inches, that make up a hundred weight of such a rope.

Suppose you desire to know how much of a seven inch rope will make a hundred weight; find 7, in the third column, under inches (or thickness of rope) and against it, in the fourth column, you find 9 5 6, which shews that there will be 9 fathoms, 5 feet, 6 inches required to make one hundred weight.

A TABLE,

Shewing the weight of any cable, or rope, of 120 *fathoms* in length, and for every half inch, from 3 to 25 inches, in circumference.

Inches.	Cwt.	Qrs.	Inches.	Cwt.	Qrs.	Inches.	Cwt.	Qrs.	Inches.	Cwt.	Qrs.	Inches.	Cwt.	Qrs.
3	2	1	7	12	1	11	30	0	15½	60	0	20	100	0
3½	3	0	7½	14	0	11½	33	0	16	64	0	20½	105	0
4	4	0	8	16	0	12	36	0	16½	68	0	21	110	1
4½	5	0	8½	18	0	12½	39	0	17	72	1	21½	115	2
5	6	1	9	20	1	13	42	1	17½	76	2	22	121	0
5½	7	2	9½	22	2	13½	45	2	18	81	0	22½	126	2
6	9	0	10	25	0	14	49	0	18½	85	2	23	132	1
6½	10	2	10½	27	2	14½	52	2	19	90	1	23½	138	0
						15	56	1	19½	95	0	24	144	0

USE OF THE TABLE.

The first column, marked inches, is the thickness or circumference of the cable, to every half inch, from 3 to 24 inches; the second, marked Cwt. Qrs. for the hundred weight and quarters that it will weigh, if 120 fathoms in length. For instance: suppose it be a cable of 14½ inches; look against 14½, and you will find, in the next column to the right hand, 52Cwt. 2qrs., which shews that 120 fathoms, of 14½ inches circumference, will weigh the above weight. In the same manner, the weight of any other is known; and by proportion, any length can be ascertained.

EXAMPLE.

A ship was brought to anchor in a gale of wind; but the gale increasing, it was thought safest to cut the cables. They did cut them; 75 fathoms of 16 inches. and 50 fathoms of 12 inches, were lost. Required, the amount of loss, reckoning the price at $14 per Cwt.

120 fath. 16 in.	64 cwt.	120 fath. 12 in.	36 cwt.
60 do.	32	40 do.	12
15 do.	8	10 do.	3
75 fath. weight	40	50 fath. weight	15
50 do.	15		

55 cwt. at $14=770.

One third deducted, 256.66⅔

$513.33⅓ Ans.

DIVISION OF DUODECIMALS.

EXAMPLE.

Divide $11F.\ 7I.\ 9''\ 11'''\ 6''''$ by $1F.\ 7I.\ 3''$.

$$\begin{array}{r} 1\ 7\ 3)11\ 7\ 9\ 11\ 6(7F.\ 3I.\ 2''.\ \text{Ans.} \\ 11\ 2\ 9 \qquad\qquad\qquad \\ \hline 5\ 0\ 11 \qquad\qquad \\ 4\ 9\ 9 \qquad\qquad \\ \hline 3\ 2\ 6 \qquad \\ 3\ 2\ 6 \qquad \end{array}$$

Note. This specimen in division, is supposed to be sufficient to illustrate that part.

Some authors give decimal tables of seconds, and inches and seconds, by which multiplications can be decimally performed; but not considering it of sufficient importance to require a table, I have omitted it.

SEXCESSIMALS.

Sexcessimals are the parts of an hour; as minutes, seconds, thirds, fourths, &c.; and the ratio of those parts to each other, is easily discovered from the following statement, viz. that

60 fourths make 1 third.
3600 fourths, or 60 thirds, 1 *second.*
216000 fourths, or 60 seconds, 1 minute.
12960000 fourths, or 60 minutes, 1 hour.

The ratio of the fourths in a third, is to the ratio of the fourths in a fourth, as 60 to 1=60; for $\frac{60}{1}=60$. The ratio of the fourths in a second, is to the ratio of the fourths in a third, as 3600 to 60=60; for $\frac{3600}{60}=60$, &c. Hence, the ratio is 60, (composed of *sex*, 6, and *decem*, 10;) therefore, when we use calculations where fourths, thirds, seconds, and minutes are concerned, we must carry by 60.

Sexcessimals are of use in various astronomical computations relating to time.

The demonstration being previously given, in the table of Motion, we shall omit it in this place.

EXAMPLES.

If two places differ in longitude 2° 12′, what is the difference of time?

```
 °  ′  ″  ‴
 2 12 00 00
  *3 59 20
  --------
   0  0  4
   0  0 40
   0 11 48
   1 58 00
   0 36 00
   6 00 00
  --------
   8 46 32 Ans.
```

$12' \times 20''' = 240'''' = 4'''$

$2^\circ \times 20''' = 40'''$

$12' \times 59'' = 708''' = 11''\ 48'''$

$2^\circ \times 59'' = 118'' = 1'\ 58''$

$12' \times 3' = 36''$

$2^\circ \times 3' = 6'$

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* The time in which the sun passes through one degree.

Two places differ in longitude 31° 27′ 30″; what is the difference, in time, of the sun's coming to the meridian of those places, the sun passing through 15° in an hour?

$$\begin{array}{rrr} 31 & 37 & 30 \\ & 4 & 00^{*} \\ \hline 2 & 6 & 30 \end{array}\ \text{Ans.}$$

$$\begin{array}{l} 30'' \times 4' = 120''' = 2'' \\ 36' \times 4' = 148'' = 2'\ 28'' \\ 31^\circ \times 4' = 124' = 2^\circ\ 4' \\ \hphantom{31^\circ \times 4' = 124' = 2^\circ\ } 2'\ 84'' \\ \hphantom{31^\circ \times 4' = 124' = 2^\circ\ 2'\ 8} 2'' \\ \hline \text{Ans. } 2\quad 6\quad 30 \end{array}$$

These two examples of sexcessimals being worked, are sufficient to shew how all similar cases should be wrought.

Multiplication of pounds, shillings, and pence, by pounds, shillings, and pence, is nearly allied to the above, although it does not come under the denomination of either Decimals, Duodecimals, or Sexcessimals.

EXAMPLES.

1. Multiply 3£ 6*s*. 8*d*. by 2£ 5*s*. 7*d*.

$$\begin{array}{ccc} 3 & 6 & 8 \\ 2 & 5 & 7 \\ \hline \end{array}$$

$$\left.\begin{array}{lrr} \pounds3 \times 2\pounds = \ 6\pounds = 6 & 0 & 0 \\ s6 \times 2\pounds = 12s. = & 12 & 0 \\ d8 \times 2\pounds = 16d. = & 1 & 4 \\ \pounds3 \times 5s. = 15s. = & 15 & 0 \\ s6 \times 5s. = \frac{30}{20}s. = & 1 & 6 \\ d8 \times 5s. = \frac{40}{20}d. = & 0 & 2 \\ \pounds3 \times 7d. = 21d. = & 1 & 9 \\ s6 \times 7d. = \frac{42}{20}d. = & 0 & 2\frac{1}{10} \\ d8 \times 7d. = \frac{7}{30}d. = & & \frac{7}{30} \end{array}\right\} \begin{array}{ccc} \pounds & s. & d. \\ =7 & 11 & 11\frac{1}{3} \end{array}\ \text{An}$$

$3\ 6\ 8 = \frac{10}{3}\pounds$

$2\ 5\ 7 = \frac{547}{240}$, and $\frac{10}{3} \times \frac{547}{240} = \frac{5470}{720} = 7\ 11\ 11\frac{1}{3}$ Ans.

---

* In 4 minutes of a solar day, or day of 24 hours, the sun passes one degree.

2. A, B, and C bought a drove of sheep in company; A paid *14£ 5s.*, B, 13£ 10*s.*, and C 11£ *5s.*; they agreed to dispose of them at market; that each man should take 18*s.* as pay for his time, &c., and that the remainder should be divided in proportion to their several stocks. At the close of the sale, they found themselves possessed of 46£ *5s.*; what was each man's gain, exclusive of the pay for his time?

$$\left.\begin{array}{rr} £ & s. \\ 14 & 5 \\ 13 & 10 \\ 11 & 5 \end{array}\right\} = £39, \text{ and } 46£\ 5s.—39£=7£\ 5s., \text{ and}$$

7£ 5*s.*—$\overline{18s.\times 3}$=4£ 11*s.* whole gain,
and 4 11÷39=2*s.* 4*d.* gain per £.

Therefore,

£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	Proof.
14	5	0	13	10	0	11	5	0	1 13 3
	2	4		2	4		2	4	1 11 6
									1 6 3
1	8	6	1	7	0	1	2	6	
	4	9		4	6		3	9	£4 11 0
£1	13	3	£1	11	6	£1	6	3	

From the above it appears, that pounds multiplied by pounds give pounds, pounds multiplied by shillings give shillings, pence multiplied by pounds give pence, and farthings multiplied by pounds give farthings. Shillings multiplied by shillings give twentieths of a shilling, shillings multiplied by pence give twentieths of a penny, and pence multiplied by pence give 240ths of a penny, &c.

Multiply 17£ 17*s.* 11$\frac{3}{4}$*d.* by 7£ 7*s.* 11$\frac{3}{4}$*d.*

ANNUITIES.

The following essay is intended to illustrate the doctrine of *simple interest;* the present worth of *annuities*, or *pensions*, &c.; *annuities*, *leases*, &c. taken in reversion; *rebate* and *discount; compound interest; annuities* and *pensions in arrears*, computed by compound interest; present worth of *annuities*, *pensions*, &c., computed in the same way; *annuities*, *leases*, &c., taken in *reversion*, and purchasing freehold estates, in *reversion.* Each case of which shall be shewn by proper formulas, derived from first principles; beginning first with

SIMPLE INTEREST.

If p = principal,
t = time,
r = ratio of the rate per cent,
a = the amount.

If r represent the rate of interest of £1 or $1 for 1 year, then $1 : r :: p : pr$ = the interest of p for 1 year; and $1 : rp :: t : prt$ = the interest for the time t, and $ptr+p=a$, the amount at the end of the time t. Wherefore, by the resolution of equations, we have,

1. $ptr+p=a.$

2. $\frac{a}{tr+1}=p.$

3. $\frac{a-p}{pt}=r.$

4. $\frac{a-p}{pr}=t.$

If p = $100, $t=9$ years, $r=.06$, and a = $154; then the four algebraic formulas, or equations, by substitution, become of the following forms, viz.

1. $100\times 9\times .06+100=154.$ am't

2. $\frac{154}{9\times .06+1}=100.$ principal.

3. $\frac{154-100}{100\times 9}=.06.$ ratio.

4. $\frac{154-100}{100\times .06}=9.$ time.

CASE 1. EXAMPLE 1.

What sum will \$100 amount to in 10 years, at 6 per cent per annum?

By theorem 1. $100\times10\times.06+100=\$160.$ Ans.

CASE 2. EXAMPLE 2.

What principal will amount to \$160 in 10 years, at 6 per cent per annum?

By theorem 2. $\dfrac{160}{10\times.06+1}=100.$ Ans.

CASE 3. EXAMPLE 3.

At what rate per cent will \$100 amount to \$160, in 10 years?

By theorem 3. $\dfrac{160-100}{100\times10}=.06.$ Ans.

CASE 4. EXAMPLE 4.

In what time will \$100 amount to \$160, at 6 per cent per annum?

By theorem 4. $\dfrac{160-100}{100\times.06}=10.$ Ans.

NOTE. The above examples are sufficient to direct how every thing relative to the rules arising from the four preceding formulas may be done, let time, rate per cent, or whatever else, be varied as they may.

ANNUITIES, OR PENSIONS, IN ARREARS.

If $u=$ annuity, or annual pension,
$r=$ ratio,
$t=$ time,
$a=$ amount,
$o=$ interest due at 1 year's end,
$ru=$ interest due at 2 year's end,
$2ru=$ interest due at 3 year's end, &c.
$\overline{t-1}\times ru=$ interest for t years,

And $0+1+2+3\ldots$ to $t-1\times$ into $ru+tu=a$ $0+1+2+3\ldots t-1=\frac{t\times t-1}{2}$. Wherefore,

1. $\frac{t^2u-tu}{2}\times r\times tu=a$. And by the resolution of equations,

2. $\frac{2a}{t^2r-tr+2t}=u$.

3. $\frac{2a-2tu}{t^2u-tu}=r$.

4. $\sqrt{\frac{8ar+4u-4ur+ur^2}{4ur^2}}-\frac{2-r}{2r}=t$.

CASE 1. EXAMPLE 1.

If an annuity of $100 *per annum, be forborne* 10 *years, what will it amount to in that time, at* 6 *per cent?*

By theorem 1.*

$\frac{100\times100-10\times100}{2}\times.06+10\times100=1270$. An.

CASE 2. EXAMPLE 2.

If the amount of an annuity for 10 *years, at* 6 *per cent, be* $1270, *what is the annuity?*

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* Note. When the annuities, or rents, are to be paid by half yearly, or quarterly payments, as most generally they are; then, for half yearly payments, take (always) half of the ratio, half the yearly rent, and twiee the number of years; that is, reduce the years to half years.

For quarterly payments, take fourth part of the ratio, fourth part of the yearly rent, and four times the number of years; that is, reduce the years to quarters, and work as before.

Note. Half yearly and quarterly payments will be found to be most advantageous, or to be more so than yearly payments. Generally, the oftener payments are made, (if daily,) so much the more profitable to the pensioner.

By theorem 2.

$$\frac{1270\times 2}{10\times 10\times .06-10\times .06+2\times 10}=100. \text{ Ans.}$$

CASE 3. EXAMPLE 3.

If an annuity of $100 per annum, amounts to $1270 in 10 years; I demand the rate per cent.

By theorem 3. $\frac{1270\times 2-100\times 10\times 2}{10\times 10\times 100-100\times 10}=.06.$ Ans.

NOTE. When the payments are half yearly, 1270×4—100×10×4 must be the dividend, and when quarterly, 1270×8—100×10×8, (allowing every thing to be the same.)

CASE 4. EXAMPLE 4.

In what time will $100 per annum amount to $1270, forborne, at 6 per cent?

By theorem 4.

$$\sqrt{\frac{1270\times 8\times .06+100\times 4-100\times 4\times .06+100\times .06\times .06}{100\times 4\times .06\times .06}}-\frac{2-.06}{2\times .06}=10. \text{ Ans.}$$

PRESENT WORTH OF ANNUITIES, OR PENSIONS, &c.

NOTE. p represents the present worth, and u, t, and r, as before.

From the preceding principles, and by the resolution of equations, we derive the following formulas, viz.

1. $\frac{t^2r-tr+2t}{2tr+2}\times u=p.$

2. $\frac{tr+1}{t^2r-tr+2t}\times 2p=u.$

3. $\frac{2tu-2p}{2pt-\overline{t^2u-ut}}=r.$

4. $\sqrt{\frac{2p}{ur}+\left(\frac{2u-2pr-ur}{2ur}\right)^2}-\frac{2u-2pr-ur}{2ur}=t.$

CASE 1. EXAMPLE 1.

What is the present worth of $100 *per annum, to continue* 10 *years, at* 6 *per cent?*

By theorem 1.

$$\frac{10\times10\times.06-10\times.06+10\times2}{2\times10\times.06+2}=\$793.75.\ \text{Ans.}$$

NOTE. The same caution must be observed here, for half yearly and quarterly payments, as in case 1, in annuities and pensions in arrears.

CASE 2. EXAMPLE 2.

There is the lease of a house 10 *years to come; I demand the yearly rent, when the present worth, at* 6 *per cent, is* $793.75?

By theorem 2.

$$\frac{10\times.06+1}{10\times10\times.06-10\times.06+10\times2}\times793.75\times2=$$
100? Ans.

NOTE. When the payments to be made, are half yearly, you must multiply by 793.75×4, and for quarterly payments, by 793.75×8, (allowing every thing to be the same.)

CASE 3. EXAMPLE 3.

I demand at what rate per cent will the yearly rent of $100, *to continue* 10 *years, produce the present worth of* $793.75.

By theorem 3.*

$$\frac{2\times10\times100-793.75\times2}{2\times10\times793.73-10\times10\times100-100\times10}=.06.$$

---

* NOTE. If the payments are to be half yearly, or quarterly; then, for half yearly payments, take half of the annuity, or yearly rent, and twice the number of years, that is, reduce the years into half years, and the quotient of the upper part divided by the lower, will be the ratio

CASE 4. EXAMPLE 4.

If $100 yearly rent produce the present worth of $793.75, at 6 per cent, what is the time of its continuance?

By theorem 4.*

$$\sqrt{\frac{793.75\times2}{100\times.06}+\left(\frac{100\times2-793.75\times2\times.06-100\times.06}{100\times2\times.06}\right)^2}-\frac{100\times2-793.75\times.06-100\times.06}{100\times2\times.06}=10.\ \text{Ans.}$$

ANNUITIES, LEASES, &c. TAKEN IN REVERSION.

First, by theorem 1, of annuities and pensions, &c. we have $\frac{t^2r-tr+2t}{2tr+2}\times2=p$.

Now, if p be changed into a, then,

$$\frac{a}{tr+1}=P,\ \text{or the principal.}$$

EXAMPLE.

What is the present worth of a lease of $100 per annum, to continue 10 years; but is not to commence till the end of 2 years?

By the first part of theorem above,

$$\frac{10\times10\times.06-10\times.06+2\times10}{2\times10\times.06+2}\times100=793.75$$

---

of half the rate per cent. For quarterly payments, take a fourth part of the annuity, or yearly rent, and four times the number of years, that is, reduce the years into quarters; and then the quotient of the upper part divided by the lower, will be the ratio of a fourth part of the rate per cent.

* If the payments are to be half yearly, then u will be equal half of the given lease, pension, &c., and r will be equal half of the given rate, and t, which is required, will be equal the number of payments, or half years.

If the payments are to be quarterly, then u will be a fourth part of the given lease, pension, &c., and r will be a fourth part of the ratio of the given rate, and t will be the number of quarterly payments.

Now, making $p=a$; we have,

$$\frac{793.75}{2\times.06+1}=708.7053+\text{ Ans.}$$

For, 708.7053+ put to interest two years, (the time elapsed before the lease commenced,) at the given rate, will give $793.75; which proves the answer to be true.

TO FIND THE YEARLY INCOME OF AN ANNUITY, &c. IN REVERSION.

By formula 1, in simple interest, $ptr+p=a$.

In numbers, $708.7053\times.06\times2+708.7053=793.75$ = amount of 708.7053+ for 2 years = the time before the reversion.

Again, per formula in case 2 of the present worth of annuities, or pensions, &c. $\frac{rt+1}{t^2r-tr+2t}\times2p=u$.

We get in numbers, $\frac{10\times.06+1}{100\times.06-10\times.06+10\times2}\times793.75\times2=\frac{2540}{25.4}=100$. Ans.

Or, $\frac{\overline{10\times.06+1}\times\overline{793.75\times2}}{100\times.06-10\times.06+10\times2}=\frac{2540}{25.4}=100$.

Note. By referring to the preceding theorems, any case of a similar kind (however varied) may be easily resolved.

REBATE OR DISCOUNT.

By the rule of discount, if 1 = year, t = any time, r = rate per cent, s = sum to be discounted, and p = present worth, $1 : t :: r : tr$ = a fourth number.

Again, $tr+1 : 1 :: s : \frac{s}{tr+1}=p$, the presen worth.

By theorem 2.

$$\frac{126.247696}{1.26247696}=\frac{126.247696}{\overline{1.06}\,|^4}=100.\ \text{Ans.}$$

CASE 3. EXAMPLE 3.

In what time will $100 *amount to* $126.247696, *at* 6 *per cent?*

By theorem 3.

$$\frac{126.247696}{100}=1.26247696=\overline{1.06}\,|^4,\ \text{and}$$

$$\left.\begin{array}{r} 1.06)1.26247696 \\ \hline 1.06)1191016 \\ \hline 1.06)11236 \\ \hline 1.06)106 \\ 106 \\ \hline \end{array}\right\}$$

Here, because it required four divisions, before it divided even without a remainder, the time is 4 years. Ans.

$$\text{Or, } \frac{2.1012236-2.0000000}{.0253059}=4.\ \text{Ans.}$$

NOTE. 2.1012236 = log. of 126.247696 = s.
2.0000000 = log. of 100. = p.
0.0253059 = log. of 1.06 = R.

CASE 4. EXAMPLE 4.

At what rate per cent will $100, *amount to* $126.247696, *in* 4 *years?*

By theorem 4.

$$\left(\frac{126.247696}{100}\right)^{\frac{1}{4}}=1.06=\text{R, and R}-1=.06=$$

$\frac{6}{100}$, or 6 for every 100 = 6 per cent. Ans.

ANNUITIES, OR PENSIONS, IN ARREARS.

If u equal the annuity, or yearly rent, t equal time of forbearance, r equal the interest of £1, or $1, for 1 year, &c. R equal $1+r$, s equal sum of all the arrears or amount.

u= money due at 1 year's end.

$2u+ru=u+Ru$=arrear at 2 year's end.

$u+uR+uR^2$=arrear at 3 year's end.

$u+uR+uR^2+uR^3$=arrear for 4 years.

$u+uR+uR^2+uR^3$ .... to uR^{t-1}=arrear for t years.

$$1+R+R^2+R^3 \ldots\ldots\ldots \text{to } R^{t-1}=\frac{R\times R^{t-1}-1}{R-1}$$

$=\frac{R^t-1}{r}$, $\frac{R^t-1}{r}\times u$=money due at the end of t years.

1. $\frac{R^t u-u}{r}=s$.

2. $\frac{rs}{R^t-1}=u$.

3. $\frac{sr+u}{u}=\frac{sr}{u}+1=R^t$ or $t=\dfrac{\text{Log. } \frac{rs}{u}+1}{\text{Log. } R}$

4. $\left(\frac{sr}{u}+1\right)^{\frac{1}{t}}=\sqrt[t]{\frac{sr}{u}+1}=R$.

$\frac{s}{u}R-R^t=\frac{s-u}{u}$, from which R may be found, and then r.

CASE 1. EXAMPLE 1.

What will an an annuity of \$100 per annum, payable quarterly, amount to in 4 years, at 6 per cent?

By theorem 1.

$$\frac{100\times 1.06\times 1.06\times 1.06\times 1.06-100}{1.06-1}=$$

$$\frac{100\times 1.06^4-100}{1.06-1}=\frac{126.247696-100}{1.06-1}=\frac{26.247696}{.06}$$

=437.4616 Ans.

By theorem 2.

$$\frac{126.247696}{1.26247696}=\frac{126.247696}{\overline{1.06}\rceil^{4}}=100.\ \text{Ans.}$$

CASE 3. EXAMPLE 3.

In what time will $100 amount to $126.247696, at 6 per cent?

By theorem 3.

$$\frac{126.247696}{100}=1.26247696=\overline{1.06}\rceil^{4},\ \text{and}$$

1.06)1.26247696

1.06)1191016

1.06)11236

1.06)106
106

Here, because it required four divisions, before it divided even without a remainder, the time is 4 years. Ans.

$$\text{Or, }\frac{2.1012236-2.0000000}{.0253059}=4.\ \text{Ans.}$$

Note. 2.1012236 = log. of 126.247696 = s.
2.0000000 = log. of 100. = p.
0.0253059 = log. of 1.06 = R.

CASE 4. EXAMPLE 4.

At what rate per cent will $100, amount to $126.247696, in 4 years?

By theorem 4.

$$\left(\frac{126.247696}{100}\right)^{\frac{1}{4}}=1.06=R,\ \text{and}\ R-1=.06=\tfrac{6}{100},$$ or 6 for every 100 = 6 per cent. Ans.

ANNUITIES, OR PENSIONS, IN ARREARS.

If u equal the annuity, or yearly rent, t equal time of forbearance, r equal the interest of £1, or $1, for 1 year, &c. R equal $1+r$, s equal sum of all the arrears or amount.

u=money due at 1 year's end.

$2u+ru=u+Ru$=arrear at 2 year's end.

$u+uR+uR^2$=arrear at 3 year's end.

$u+uR+uR^2+uR^3$=arrear for 4 years.

$u+uR+uR^2+uR^3$ .... to uR^{t-1}=arrear for t years.

$$1+R+R^2+R^3 \ldots\ldots \text{to } R^{t-1}=\frac{R\times R^{t-1}-1}{R-1}$$

$=\frac{R^t-1}{r}$, $\frac{R^t-1}{r}\times u$=money due at the end of t years.

1. $$\frac{R^t u-u}{r}=s.$$

2. $$\frac{rs}{R^t-1}=u.$$

3. $$\frac{sr+u}{u}=\frac{sr}{u}+1=R^t \text{ or } t=\frac{\text{Log. }\frac{rs}{u}+1}{\text{Log. }R}$$

4. $$\left(\frac{sr}{u}+1\right)^{\frac{1}{t}}=\sqrt[t]{\frac{sr}{u}+1}=R.$$

$\frac{s}{u}R-R^t=\frac{s-u}{u}$, from which R may be found, and then r.

CASE 1. EXAMPLE 1.

What will an an annuity of $100 per annum, payable quarterly, amount to in 4 years, at 6 per cent?

By theorem 1.

$$\frac{100\times 1.06\times 1.06\times 1.06\times 1.06-100}{1.06-1}=$$

$$\frac{100\times 1.06^4-100}{1.06-1}=\frac{126.247696-100}{1.06-1}=\frac{26.247696}{.06}$$

=437.4616 Ans.

CASE 2. EXAMPLE 2.

What annuity, being forborne 4 years, will amount to $437.4616, at 6 per cent?

By theorem 2.

$$\frac{437.4616\times.06}{\overline{1.06}|^{4}-1}=\frac{26.247696}{.26247696}=100.\ \text{Ans.}$$

CASE 3. EXAMPLE 3.

In what time will $100 per annum amount to $457.4616, allowing 6 per cent for the forbearance of payment?

By theorem 3.

$$\frac{437.4616\times.06}{100}+1=1.26247696=R^{t}$$

$$\frac{\text{And Log. of } 1.26247696=.1012236}{\text{Log. of } 1.06\quad=.0253059}=4.\ \text{Ans.}$$

$$\left.\begin{array}{r}\text{But } 1,06)1,26247696\\ \hline 1,06)1191016\\ \hline 1,06)11236\\ \hline 1,06)106\\ 106\\ \hline \end{array}\right\}$$

Here, because four divisions, by the amount of $1 for 1 year equal R, exhausts the power $R^{t}=1.26247696$, the time is 4 years.

CASE 4. EXAMPLE 4.

At what rate per cent, will an annuity of $100 per annum amount to $437.4616, in 4 years?

By theorem 4.

$$\frac{437.4616-100}{100}=3.374616=\frac{437.4616}{100}R-R^{4},\ \text{or}$$

$R^{4}-4.374616R=-3.374616$, an equation from which R is foucd $=1.06$, and $1.06-1=.06=r$, or 6 per cent. Ans.

PRESENT WORTH OF ANNUITIES, PENSIONS, &c.

If p equal present worth, u equal the annuity, t equal the time, r equal the interest of £1 or $1 for 1 year, R equal $1+r$; hence, by formula pR^t equal s, and by a preceding formula, $\frac{R^t-1}{r} \times u$ also equal s.

We have, $pR^t = \frac{R^t-1}{r} \times u$, from which, and by the resolution of equations,

$$1. \quad p = \frac{R^t-1}{rR^t} \times u = \frac{1-\frac{1}{R^t}}{r} \times u$$

$$2. \quad u = \frac{pr}{1-\frac{1}{R^t}}$$

$$3. \quad R^t = \frac{u}{u-pr}, \text{ or } t = \frac{\text{Log. } u - \text{log. } \overline{u-pr}}{\text{Log. } R}$$

$$4. \quad R^t + \frac{u}{p}R^t - R^{t+1} = \frac{u}{p},$$ an equation from which R can be found, and r, likewise. But from the third formula $R = \left(\frac{u}{u-pr}\right)^{\frac{1}{t}}$; however, as the value of r depends upon that of R^t, it had better come from the fourth formula.

CASE 1. EXAMPLE 1.

What is the yearly rent of $100, to continue 4 years, worth in ready money, at 6 per cent?

By formula 1.

$$\frac{1.06^4-1}{1.06^4 \times .06} \times 100 = \frac{1.26247696-1}{1.26247696 \times .06} \times 100 =$$

346.5105+ Ans.

CASE 2. EXAMPLE 2.

What annuity or yearly rent, to continue 4 years, may be purchased for $346.5105+ at 6 per cent?

By theorem 2.

$$\frac{346.5105\times .06}{1-\frac{1}{1.06^4}}=\frac{20.79063}{1-.7920937}=\frac{20.79063}{.2079063}=100.$$

CASE 3. EXAMPLE 3.

How long may a lease of $100 per annum, be had for $346.5105+ allowing 6 per cent to the purchaser?

By theorem 3.

$$\frac{\text{Log. }100-\text{log. }100-20.79063}{\text{Log. }1.06}=\frac{2-1.8987764}{.0253059}=$$

$$\frac{.1012236}{.0253059}=4.\ \text{Ans.}$$

CASE 4. EXAMPLE 4.

At what rate per cent per annum will an annuity of $100 amount to $346.5105+ in 4 years?

By theorem 4.

$R^{4+1}-\frac{100}{346.5106}R^t=\frac{100}{346.5105}$, an equation from which R is found, equal 1.06, and $1.06-1=r=\frac{6}{100}$ or 6 per cent. Ans.

ANNUITIES, LEASES, &c. TAKEN IN REVERSION.

If v equal time in reversion, p equal first present worth, and P equal second present worth, arising from changing $p\div R-1$ into m, t equal continuance of lease, and the other terms as before. From the nature of annuities in reversion, we have

1. $u-\frac{u}{R^t}=p$, and $p\div\overline{R-1}$ = a number, which change into m, and $\frac{m}{R^v}=P$

2. $PR^v = m$. If m be changed into p,

$$\frac{pR^t \times R - pR^t}{R^t - 1} = u.$$

Or, 1. $\frac{R^t - 1 \times u}{\overline{R-1} \times R^t \times R^v} = P$ 2. $\frac{\overline{R-1} \times R^t \times R^v P}{R^t - 1} = u.$

Again, $pR^t = \text{amount} = a$, which change into P and we have,

3. $\frac{u}{P+u-PR} = R^t$, and $t = \frac{\text{Log. } u - \text{log. } \overline{u+P-PR}}{\text{Log. } R}$

4. $R^{t+1} - \frac{R^t \times \overline{P+u}}{P} = -\frac{u}{P}$ an equation from which R can be found.

CASE 1. EXAMPLE 1.

What is the present worth of the reversion of a lease of $100 *per annum, to continue* 4 *years, but not to commence till the end of* 2 *years, allowing* 6 *per cent to the purchaser?*

By theorem 1.

$$100 - \frac{100}{1.26247696} = 20.7907, \text{ and } \frac{20.7907}{.06} =$$

$$346.511666; \text{ again } \frac{346.511666}{1.06^2} = \frac{346.511666}{1.1236} =$$

308.3941 + Ans.

CASE 2. EXAMPLE 2.

What annuity, or yearly rent, to be entered upon 2 *years hence, and then to continue* 4 *years, may be purchased for* $308.3941 *ready money, at* 6 *per cent?*

By theorem 2. $\frac{.06 \times 1.06^4 \times 1.06^2 \times 308.3941}{1.06^4 - 1} =$

$$\frac{.06 \times 1.26247696 \times 1.1236 \times 308.3941}{1.26247696 - 1} = 100. \text{ Ans.}$$

CASE 3. EXAMPLE 3.

The present worth of a certain lease in reversion, is 308 3941; *the lease is* $100 *per annum, and commences*

2 years hence, and the allowance to the purchaser is 6 per cent; I demand the time of its continuance.

By theorem 3. First, $308.3941 \times 1.06^2 = 308.3941 \times 1.1236 = 346.511666$.

Again, $\frac{100}{346.511666 + 100 - 346.511666 \times 1.06} =$ 1.26247696 and

$$\left.\begin{array}{r} 1.06)1.26247696 \\ \hline 1.06)1191017 \\ \hline 1.06)11236 \\ \hline 1.06)106 \\ 106 \\ \hline \end{array}\right\}$$

Here the number of divisions, before the dividend was exhausted, being 4, the time is 4 years.

But by logarithms,

$$\frac{\text{L. } 100 - \text{L. } \overline{346.511666 + 100 - 346.511666 \times 1.06}}{\text{Log. } 1.06}$$

$= 4$. Ans.

That is, $\frac{2 - 1.898774}{.0253059} = 4$. Ans.

CASE 4. EXAMPLE 4.

The present worth of a certain lease in reversion, is $308.3941; the lease is $100 per annum, and commences 2 years hence, to continue 4 years; required, the rate per cent.

By theorem 4.

$$\overline{1.06}\,\rceil^{6} - \frac{308.3941 + 100}{308.394} \times \overline{1.06}\,\rceil^{4} = -\frac{100}{308.3941}$$

an equation from which the amount of $1 per year is had, equal 1,06, and $1.06 - 1 = .06 = \frac{6}{100} = 6$ per cent. Ans.

PURCHASING REAL OR FREEHOLD ESTATES.

A real, or freehold estate, is such as is bought to continue forever.

To find the value of an annuity to continue forever, at a given rate of compound interest.

Let p equal present worth, u equal annuity, r equal interest of £1, or \$1, R equal $1+r$ equal principal and interest of £1, or \$1, for one year.

$p=\frac{R^t-1}{rR^t}\times u$. But since t is infinite, R^t is infinitely greater than 1, whence $R^t-1=R^t$ and $p=\left(\frac{R^t}{rR^t}\times u=\right)\frac{u}{r}$

2. $u=pr$.

3. $r=\frac{u}{p}$

CASE 1. EXAMPLE 1.

Suppose a freehold estate (or one which is bought to continue forever) of \$100 per annum; what is it worth, allowing the buyer 6 per cent for his money?

By theorem 1. $\frac{100}{1.06-1}=\$1666\frac{2}{3}$ Ans.

CASE 2. EXAMPLE 2.

If a freehold estate is bought for \$1666⅔, the allowance of 6 per cent made to the buyer; I demand the yearly rent.

By theorem 2. $1666\frac{2}{3}\times\overline{1.06-1}=\$100$. Ans.

CASE 3. EXAMPLE 3.

If a real estate of \$100 per annum, be sold for \$1666⅔, I demand the rate per cent.

By theorem 3. $\frac{1666\frac{2}{3}+100}{1666\frac{2}{3}}=1.06=$ amount of £1, or \$1, at 6 per cent, and $1.06-1=.06=\frac{6}{100}$ $=6$ per cent. Ans.

NOTE. As the time is supposed to be infinite, in such questions as the preceding, no formula is necessary to shew the value of t.

PURCHASING FREEHOLD ESTATES IN REVERSION.

From the doctrine advanced in case 1 of the purchasing real estates, &c. $\frac{u}{R-1}=p$, and $\frac{p}{R^t}=P$.

2. $PR^t=p$, and $\frac{pR^2-pR}{R}=pR-p=u$.

CASE 1. EXAMPLE 1.

If a freehold estate of $100 per annum, to commence 2 years hence, is to be sold, what is its worth, allowing the purchaser 6 per cent for his present payment?

By theorem 1.

$$\frac{100}{1.06-1}=1666\tfrac{2}{3}, \text{ and } \frac{1666\frac{2}{3}}{1.06^2}=1483.2384+ \text{ Ans.}$$

CASE 2. EXAMPLE 2.

Suppose a freehold estate, to commence 2 years hence, is sold for $1483.2384, allowing the purchaser 6 per cent for his present payment; what is the yearly income?

By theorem 2. $1483.2384\times1.06^2=1666\tfrac{2}{3}$

and $1666\tfrac{2}{3}\times1.06-1666\tfrac{2}{3}=100$. Ans.

LOGARITHMS.

As we have had occasion to use logarithms, in solving several problems in the preceding part of this work, we shall briefly show how they may be constructed, and also, something of their uses.

Logarithms are a series of numbers, so contrived, that by them the work of multiplication may be performed by addition; and the operation of division may be done by subtraction. Or, logarithms are the indices, or series of numbers in arithmetical progression, corresponding to another series of numbers in geometrical progression. Thus:

0, 1, 2, 3, 4, 5, 6, &c. Indices, or logarithms.
1, 2, 4, 8, 16, 32, 64, &c. Geom. progression.

Or,

0, 1, 2, 3, 4, 5, 6, &c. Indices, or log.
1, 3, 9, 27, 81, 243, 729, &c. Geometrical series.

Or,

0, 1, 2, 3, 4, 5, 6, In. or L.
1, 10, 100, 1000, 10000, 100000, 1000000, &c.
Geometrical series. Where the same indices serve equally for any geometrical series, or progression.

Hence it appears, that there may be as many kinds of indices, or logarithms, as there can be taken kinds of geometrical series. But the logarithms most convenient for common uses, are those adapted to a geometrical series increasing in a tenfold ratio, or progression, as in the last of the foregoing examples. In the geometrical series, 1, 10, 100, 1000, &c. if between the terms 1 and 10, the numbers 2, 3, 4, 5, 6, 7, 8, 9 were interposed, indices might also be adapted to them, in an arithmetical progression, suited to the terms interposed between 1 and 10, considered as a geometrical progression. Moreover, proper indices may be found to all the numbers that can be introduced between any two terms of the geometrical series.

But it is evident, that all the indices to the numbers under 10, must be less than 1; that is, they must be fractions. Those to the numbers between 10 and 100, must fall between 1 and 2; that is, they are mixed numbers, consisting of 1, and some fraction. Likewise, the indices to the numbers between 100 and 1000, will fall between 2 and 3; that is, they are mixed numbers, consisting of 2 and some fraction; and so of the other indices. Hereafter the integral part only of these indices, will be called the index; and the fractional part will be called the logarithm.

The computation of these fractional parts, is called *making logarithms;* and the most troublesome part of this work, is to make the logarithms of prime numbers, or those which cannot be divided by any other numbers than themselves and unity.

RULE,

For computing the logarithms of numbers.

Let the sum of its proposed number be called A. Divide 0.8685889638* by A, and reserve the quotient; divide the reserved quotient by the square of A, and reserve this quotient; divide the last reserved quotient by the square of A, reserving the quotient still; and thus proceed as long as division can be made. Write the reserved quotients orderly under each other, the first being uppermost; divide these quotients respectively by the odd numbers 1, 3, 5, 7, 9, 11, &c.; that is, divide the first reserved quotient by 1, the second by 3, the third by 5, the fourth by 7, &c., and let the quotients be written orderly under each other; add them together, and their sum will be a logarithm. To this logarithm add the logarithm of the next less number, and the sum will be the logarithm proposed.

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[*] 0.8685889638+ is the quotient of 2, by 2.302585093† which is the logarithm of 10; according to the first form of Lord Napier, the inventor of logarithms. The manner in which Napier's logarithm of 10 is found, may be seen in most books of algebra, and in systems of mathematics generally, where the analytical art is discussed.

[†] The logarithm of 10 itself, is derived from the following theorem, viz. If $10=\frac{1+e}{1-e}$, e is found $=\frac{9}{11}$; but,

$$2\times e+\frac{e^3}{3}+\frac{e^5}{5}+\frac{e^7}{7}+\frac{e^9}{9}+\frac{e^{11}}{11}+\frac{e^{13}}{13}, \text{ \&c.} = 2.3025850930$$

=Napier's Log. of 10. Brigg's Log. of 10=1.

EXAMPLE 1.

Required, the logarithm of the number 2.

Here the next less number is 1, and $2+1=3=A$, and A^2 or $3^2=9$; then,

3)0.868588964

9)9.289529654 ÷ 1 = 0.289529654

9)0.032169961 ÷ 3 = 0.010723320

9)0.003574440 ÷ 5 = 0.000714888

9)0.000397160 ÷ 7 = 0.000056737

9)0.000044129 ÷ 9 = 0.000004903

9)0.000004903 ÷ 11 = 0.000000445

9)0.000000544 ÷ 13 = 0.000000042

0.000000060 ÷ 15 = 0.000000004

To this logarithm 0.301029993
add the logarithm of 1 = 0.000000000

Their sum. 0.301029993 log. of 2

The manner in which the division is here carried on, may be readily perceived, by dividing, in the first place, the given decimal by A, and the succeeding quotients by A^2; then letting these quotients remain in their situation, as seen in the example, divide them respectively by the odd numbers, and place the new quotients in a column by themselves. By employing this process, the operation is considerably abreviated.

EXAMPLE 2.

Required the logarithm of the number 3.

Here the next less number is 2; and $3+2=5=A$, and $A^2=25$.

5)0.868588964

25)0.173717792 ÷ 1 = 0.173717792

25)0.006948711 ÷ 3 = 0.002316237

25)0.000277948 ÷ 5 = 0.000055589

25)0.000011117 ÷ 7 = 0.000001588

25)0.000000444 ÷ 9 = 0.000000049

0.000000017 ÷ 11 = 0.000000001

To this logarithm 0.176091256
add the logarithm of 2 = 0.301029993

Their sum, 0.477121249 log. of 3.

Then, because the sum of the logarithms of numbers, gives the logarithm of their product; and the difference of the logarithms, gives the logarithm of the quotient of the numbers. From the preceding logarithms, and the logarithm of 10, which is 1, a great many logarithms can be easily made, as in the following examples.

EXAMPLE 3.

Required, the logarithm of 4.

Since $2\times2=4$,
Then, to the logarithm of 2 = 0.301029993
Add the logarithm of 2 = 0.301029993

The sum equal logarithm of 4 = 0.602059986

EXAMPLE 4.

Required the logarithm of 5.

$10 \div 2 = 5$, therefore,
From the logarithm of 10=1.000000000
Subtract the logarithm of 2=0.301029993

The remainder is the log. of 5=0.698970007

EXAMPLE 5.

Required the logarithm of 6.

$6 = 3 \times 2$, therefore,
To the logarithm of 3=0.477121239
Add the logarithm of 2=0.301029993

Their sum equal logarithm of 6=0.778151232

EXAMPLE 6.

Required the logarithm of 8.

$8 = 2^3$ therefore,
Multiply the logarithm of 2=0.301029993
By 3

The product equal logarithm of 8=0.903089979

EXAMPLE 7.

Required the logarithm of 9.

$9 = 3^2$, therefore,
The logarithm of 3=0.477121239
Being multiplied by 2

The product equal the log. of 9=0.954242578

EXAMPLE 8.

Required the logarithm of 7.

Here the next less number is 6, and $6+7=13=A$, and $A^2=169$

13)0.868588964

169)0.066814535 ÷ 1 = 0.066814535

169)0.000395352 ÷ 3 = 0.000131784

169)0.000002339 ÷ 5 = 0.000000468

0.000000014 ÷ 7 = 0.000000002

To this logarithm = 0.066946789
Add the logarithm of 6 = 0.778151239

Their sum equal 0.845098028 = log. of 7.

The log.	of 12	Is equal to the sum of the log's	of 3 and 4.
	of 14		of 7 and 2.
	of 15		of 3 and 5.
	of 16		of 4 and 4.
	of 18		of 3 and 6.
	of 20		of 4 and 5.

The logarithms of the prime numbers, 11, 13, 17, 19, &c., being computed by the foregoing general rule, the logarithms of the intermediate numbers are easily found by composition and division. It may, however, be observed, that the operation is shorter in the larger prime numbers; for when any given number exceeds 400, the first quotient, being added to the logarithm of its next lesser number, will give the logarithm sought, true to 8 or 9 places; and, therefore, it will be very easy to examine any suspected logarithm in the tables.

To add and subtract by logarithms is more curious than useful, and is generally omitted in the books.

I shall, however, shew how addition and subtraction may be performed by logarithms, first, and then go on to give various examples in the arithmetic of logarithms.

If a equal the greater of any two numbers to be added or subtracted, and b equal the lesser,

$$\left.\begin{array}{ll}\text{Then} & \frac{a}{b}+1\times b=a+b\\ \text{And} & \frac{a}{b}-1\times b=a-b\end{array}\right\}$$

From these considerations the following rule results:

From the logarithm of the greater, subtract the logarithm of the lesser, and find the number corresponding to the remainder; then, if the original numbers are to be added, add 1 to the number last found; if to be subtracted, subtract 1; and the logarithm of the remainder, thus increased, or diminished, added to the logarithm of the lesser original number, will give the logarithm of the sum, or difference.

EXAMPLE 1.

Add 350 its log. 2.5440680
To 240 its log. 2.3802112

Number 1.458 log. = 0.1638568
1

Sum 2.458 log. = 0.3905819
Add log. of lesser 240 = 2.3802112

Sum answering to log. of 350 + 240 = 590 Ans. } 2.7707931

EXAMPLE 2.

Subtract 240 from 350 its log. 2.5440680
240 its log. 2.3802112

1.4583 number answering to diff. logs.0. 1638568
1

.4583 difference, its log. = $-\overline{9}.6698939$
Add log. of 240, with its index + 10 = 12.3802112
The number 110 answering to the opposite log. is equal to the required difference. } 2.0411051

This log. is not very exact, being only true to 3 places of decimals.

Or, if log. $\frac{4583}{10000}$ be taken, which is equal to the log. of .4583, it is done thus:

Log. of 4583 = 3.6608939
Minus log. of 10000 } 4.0000000

= —9.6608939 = log. of .4583 equal
—1.6608939
To which add log, of 240 = 2.3802112

Log. of difference = (110.) 2.0411051

MULTIPLICATION OF LOGARITHMS.

RULE.

Add together the logarithms answering to the given numbers, (which must be previously sought for in the table;) the sum of these logarithms will answer to the product required.

Whatever is carried from the decimal part of the logarithm, is to be added to the affirmative indices, but subtracted from the negative. Likewise, the indices must be added together, when they are all of the same kind; that is, when they are all affirmative, or all negative; but when they are of different kinds, the difference must be found, which will be of the same denomination with the greater.

EXAMPLE 1.

Required, the product of 86.25 multiplied by 6.48.

Log. of 86.25=1.935759

Log. of 6.48=0.811575

Product=558.9 2.747334

EXAMPLE 2.

Required, the product of 46.75 and .3275.

Log. of 46.75=1.669782

Log. of .3275=1.515211

Product=15.31+ 1.184993

EXAMPLE 3.

Required, the product of 3.768, 2.053, and .007693.

Log. of 3.768=0.576111

Log. of 2.053=0.312389

Log. of .007693=—3.886096

Product of .05951+ —2.774596

EXAMPLE 4.

Required, the product of 27.63, 1.859, .7258, and .03591.

Log. of 27.63=1.441381

Log. of 1.859=0.269279

Log. of .7258=—1.860817

Log. of .03591=—2.555215

Product nearly=1.339=0.126692

DIVISION BY LOGARITHMS.

RULE.

From the logarithm of the dividend subtract the logarithm of the divisor, and the remainder will be the logarithm of the quotient. In this operation, the index of the divisor must be changed from affirmative to negative, or from negative to affirmative; and then the difference of the affirmative and negative indices must be taken for *the index to the logarithm of* the quotient. Likewise, when one has been borrowed in the left hand place of the decimal part of the logarithm, add it to the index of the divisor, if affirmative, but subtract it, if negative; and let the index thence arising, be changed, and worked with as before.

EXAMPLE 1.

Divide 558.9 by 6.48.

Log. of 558.9=2.747334
Log. of 6.48=0.811575

Quotient, 86.25=1.935759

EXAMPLE 2.

Divide 15.31 by 46.75.

Log. of 15.31=1.184975
Log. of 46.75=1.669782

Quotient, .3275= —1.515193

EXAMPLE 3.

Divide .05951 by .007693.

Log. of .05951= —2.774590
Log. of .007693= —3.886096

Quotient, 7.735=0.888494

EXAMPLE 4.

Divide .6651 by 22.5.

Log. of .6651= —1.822887
Log. of 22.5= 1.352183

Quotient, .02956= —2.470704

PROPORTION,

OR THE RULE OF THREE IN LOGARITHMS.

Having stated the three given terms, according to the rule in common arithmetic, write them orderly under one another, with the signs of proportion; then add the logarithms of the second and third terms together, and from their sum subtract the logarithm of the first term, and the remainder will be the logarithm of the fourth term, or answer. Or, add together the arithmetical complement* of the logarithm of the first term, and the logarithms of the second and third terms; the sum, rejecting 10 from the index, will be the logarithm of the fourth term, or the term required.

EXAMPLES.

If a clock gain 14 seconds in 5 days 18 hours, how much will it gain in 17 days 15 hours?

5.75 days : Log.=0.759668
17.625 : : Log.=1.246129
14 sec. : Log.=1.146128

2.392257
0.759668 upper log.

Ans. 42."91=1.632589

---

* Arithmetical complement, is the common logarithm subtracted from 10 or 20.

Or thus, 5.75 days : Arith. co. log.=9.240332

17.625 : : log.=1.246129

14 seconds : log.=1.146128

Ans. 42.″91 as before. 1.632589

2. Find a fourth proportional to 98.85, 1.969, and 347.2

98.45 : log.=1.993216

347.2 : : log.=2.540580

1.969 : log.=0.294246

2.834826

1.993216

Ans. 6.944=0.841610

3. What number will have the same proportion to .8528 as .3275 has to .0131.

.0131 : log.=—2.117271

.3275 : : log.=—1.515211

.8538 : log.=—1.931356

—3.446567

—2.117271 upper log.

Ans. 21.35=1.329296

4. Required, a third proportional number to 9.642 and 4.821.

9.642 : log.=0.984167

4.821 : : log.=0.683137

4.821 : log.=0.683137

1.366274

0.984167 upper log.

Ans. 2.411=0.382107

INVOLUTION.

To find any power of a given number by logarithms.

RULE.

Multiply the logarithm of the given number by the index of the proposed power, and the product will be the logarithm, whose natural number is the power required.

When a negative index is thus multiplied, its product is negative; but what was carried from the decimal part of the logarithm, must be affirmative; consequently, the difference is the index of the product, which difference must be considered of the same kind with the greater, or that which was made the minuend.

EXAMPLES.

1. What is the second power of 3.874?

log. of 3.874=0.588160
Index 2

Power required, 15.01=1.176320

2. Required, the third power of the number 2.768.

Log. of 2.768=0.442166
3

Ans. 21.21=1.326498

3. Required, the second power of the number .2857.

Log. of .2857= —1.455910
Index 2

Ans. .08162= —2.911820

4. Required, the third power of the number .7916.

Log. of .7916= —1.898506
3

Ans. .4971= —1.695518

EVOLUTION.

To extract any proposed root of a given number, by logarithms,

RULE.

Find the logarithm of the given number, and divide it by the index of the proposed root; the quotient is a logarithm, whose natural number is the root required.

When the index of the logarithm to be divided is negative, and does not exactly contain the divisor without some remainder, increase the index by such a number as will make it exactly divisible by the index, carrying the units borrowed as so many tens to the left hand place of the decimal, and then divide as in whole numbers.

EXAMPLES.

1. Required, the square root of 847.

Index 2)2.927883=log. of 847.

1.463941=quotient=log. of 29.103+ An.

2. Required, the cube root of 847.

Index 3)2.927883

0.975961=quotient=log. of 9.462. Ans. nearly.

3. Required, the square root of .093.

Index 2)—2.968483=log. of .093

—1.484241=quot.=log. of .304959, Ans.

4. Required, the cube root of 12345.

Index 3)4.091491=log. of 12345.

1.363830=quotient=log. of 23.116. Ans.

To find the logarithm of any given number exceeding those in the table,

RULE.

If the given number exceeds four figures, find the difference between the logarithms answering to the first four figures of the given number, and the next following logarithm; multiply this difference by the remaining figures in the given number, point off as many figures to the right hand as there are in the multiplier, and the remainder, added to the logarithm answering to the first four figures, will be the required logarithm, nearly.

Thus: to find the logarithm of 7385824.

The log. of the first four figures, viz. 7385=3.868350
The next greater log.=3.868409

59
Multiplied by the remaining figures 824

236
118
472

48|616

First logarithm, viz. log. of 7385=3,868350

Log. of 7385824= 6.868398
738582.4= 5.868398
73858.24= 4.868398
7385.824= 3.868398
738.5824= 2.868398
73.85824= 1.868398
7.385824= 0.868398
.7385824= —1.868398
.07385824=&c—2,868398

The logarithms are the same for decimal numbers as for whole, except that the indices are negative.

To find the number answering to any given log.,

RULE.

If the given logarithm cannot be exactly found in the table, and if more than four figures be wanted in the corresponding natural number; then find the difference between the given and the next less logarithms, to which annex as many cyphers as there are figures required above four in the natural number; which divide by the difference between the next less and next greater logarithms, and the *quotient* annexed to the four figures formerly found, will give the required natural number.

Thus: to find the natural number of the log. 4.828991.
The next less logarithm is 3.828982, which gives 6735.
The next greater logarithm is 3.829046
3.828982 less log.

Difference 64

Next less log.=3.828982
Given log.=3.828991

```
64)90(1.4
   64
   --
   260
   256
   ---
     4
```

Therefore, 1.4 being annexed to 6735, will give 67351.4 for the number required.

The logarithm of a vulgar fraction is found by taking out the logarithms of numerator and denominator, and taking their difference; the remainder is the logarithm required.

If $3^n=19$ was given, the value of n is easily found.

Thus, viz. $n \times \text{log. } 3 = \text{log. } 19$, and $\dfrac{\text{log. } 19}{\text{log. } 3} = n$.

But $\dfrac{\text{log. } 19 = 1.278754}{\text{log. } 3 = {}^{*}0.477121} = 2.680146 = n$;

Therefore, $3^{2.680146} = 19$.

---

* Log. 3 : log. 19 :: 1 : 2.680145.

Given, $13^{1.055701}$ to find the number answering to its power. $1.055701\times\log.13=1.176091=\log.15$; therefore, $13^{1.055701}=15$.

If $19^{\frac{1}{n}}=3$, was given to find the value of n, $\frac{1}{n}\times\log.19=\log.3$; that is, $\frac{\log.19}{n}=\log.3$, and $\log.19=\log.3\times n$; therefore, $\frac{\log.19}{\log.3}=2.680146=n$, and $\frac{1}{19^{2.680146}}=3$. From which we gather the following rules, viz.

1. The logarithm of the root of any power, multiplied by the index of such power, is equal to the logarithm of the power itself; from which, by having any root and its exponent given, its corresponding power can from thence be ascertained.

2. If any power whatever, and the index of its root, be given, its corresponding root can from thence be ascertained.

NOTE. The above examples prove that the indices of all numbers are logarithms.

By the above it is found that

$1=2^{0}=2^{0}$ In like manner $1=10^{0}$
$2=2^{1}=2^{1}$ $2=10^{.30103}$
$3=2^{\frac{158496}{100000}}=2^{1.58496}$ $3=10^{.47712}$
$4=2^{2}=2^{2}$ $4=10^{.60236}$
$5=2^{\frac{232193}{100000}}=2^{2.32193}$ $5=10^{.69897}$
$6=2^{\frac{258496}{100000}}=2^{2.58496}$ $6=10^{.77815}$
$7=2^{\frac{280735}{100000}}=2^{2.80735}$ $7=10^{.84510}$
$8=2^{3}=2^{3}$ $8=10^{.90309}$
$9=2^{\frac{316992}{100000}}=2^{3.16992}$ $9=10^{.95424}$
$10=2^{\frac{332193}{100000}}=2^{3.32193}$ $10=10^{1}$

We may therefore infer, that all positive numbers whatever, may be regarded as powers of any given positive number; or at least such powers of it may be found, as shall differ from them by quantities less than any which can be assigned.

Many pretty things might be shewn of logarithms, which the bounds of the present essay will not justify me in doing. I shall conclude on this head, by saying, that the convenience and indispensable use of logarithms, not only of this sort, but of many others, makes it a subject well worth the time and attention of students: without a knowledge of which, *they* never can be mathematicians.

DIVERTING EXERCISES.

1. There is a certain number, which being divided by 11, the quotient resulting multiplied by 5, that product divided by 4, from the quotient subtract 75, to the remainder add 40, and half the sum shall make 45. Required, the number.

$$45\times2-40+75\times4\div5\times11=1100.\ \text{Ans.}$$

2. A can do a piece of work, alone, in 7 days, and B, in 11; in what time could they both together do the work?

Now, because the ratio of B's work to that of A's, is $\frac{7}{11}$ (for 7 : 11 is in the ratio of $\frac{7}{11}$,) if A work 1, B will work $\frac{7}{11}$ of 1; therefore, $1+\frac{7}{11}=$ what both could do in a day, if working together $=\frac{18}{11}$; therefore, $\frac{18}{11}w.:1w.::7d.:4\frac{5}{18}d.$ Ans.

$$\begin{array}{l} \\ \text{The same} \\ \text{otherwise} \end{array}\left\}\begin{array}{ccccccc} d & & d & & w & & w \\ 7 & : & 1 & :: & 1 & : & \frac{1}{7} \\ 11 & : & 1 & :: & 1 & : & \frac{1}{11} \end{array}\right\}\text{then } \overset{w}{\frac{1}{7}}+\overset{w}{\frac{1}{11}}=\overset{w}{\frac{18}{77}}$$

Therefore, $\frac{18}{77}w:1w::1d:4\frac{5}{13}.$ Ans.

But $7+11:7\times11::1:4\frac{5}{18}$ same as before.

3. A sheep-fold was robbed three nights successively; the first night half the sheep were stolen and half a sheep more; the second, half the remainder were lost and half a sheep more; the last night, they took half what were left and half a sheep more; by which time they were reduced to 20. How many were there at first? Ans. 167.

4. A and B together can build a boat in 20 days, with the assistance of C they can do it in 12; in what time would C do it by himself?

$$\begin{array}{ccccccc} d & & d & & w & & w \\ 20 & : & 1 & :: & 1 & : & \frac{1}{20} \\ 12 & : & 1 & :: & 1 & : & \frac{1}{12} \\ & & & & w & & w \end{array} \Bigg\} \text{then } \frac{1}{12}-\frac{1}{20}=\frac{8}{240}=\frac{1}{30} \quad \begin{array}{cc} \\ \\ d & d \end{array}$$

Therefore, 1 : 30 :: 1 : 30 Ans.

But 20—12 : 20×12 :: 1 : 30 the same as before.

5. A can do a piece of work in 15 days, and A and B together in 8 days; in what time can B do it alone?

$$\begin{array}{ccccccc} d & & d & & w & & w \\ 15 & : & 1 & :: & 1 & : & \frac{1}{15} \\ 8 & : & 1 & :: & 1 & : & \frac{1}{8} \\ & & w & & w & & \end{array} \Bigg\} \text{then } \frac{1}{8}-\frac{1}{15}=\frac{7}{120} \quad \begin{array}{cc} \\ \\ d & d \end{array}$$

And as 7 : 120 :: 1 : 17$\frac{1}{7}$ Ans.

But 15—8 : 15×8 :: 1 : 17$\frac{1}{7}$ Ans.

6. A cistern, for water, has two cocks to supply it; by the first, it may be filled in 45 minutes, and by the second in 55 minutes; it has likewise a discharging cock, by which it may, when full, be emptied in 30 minutes; now if these three cocks be all left open when the water comes in, in what time will the cistern be filled?

$$\left.\begin{array}{l} \;\,' \quad\;\;\, ' \qquad\quad \textit{cis.} \quad \textit{cistern.} \\ 45 : 60 :: 1 : 1.3333 \\ 55 : 60 :: 1 : 1.0909 \end{array}\right\} \overset{\textit{cistern.}}{=2.4242} \text{ and } 30 : 60 :: 1 : 2$$

2.4242—2=.4232 gain in an hour.

$$\begin{array}{l} \qquad\qquad\quad\; \textit{cistern.} \;\; \textit{cis.} \quad\;\; h \quad\;\; h \;\; ' \;\; '' \\ \text{Therefore, } .4232 : 1 :: 1 : 1.21.26\tfrac{1}{2} \end{array}$$

7. A, B, and C can complete a piece of work in 15 days, A can do it alone in 23 days, and B in 37 days; in what time can C do it by himself?

$$\left\{\begin{array}{l} \overset{d}{15} : \overset{d}{1} :: \overset{w}{1} : \overset{w}{\tfrac{1}{15}} \\ \hline 23 : 1 :: 1 : \tfrac{1}{23} \\ 37 : 1 :: 1 : \tfrac{7}{37} \end{array}\right\} \text{then } \overset{w}{\tfrac{1}{15}} - \overset{w}{\tfrac{1}{23}} + \overset{w}{\tfrac{1}{37}} = \overset{w}{\tfrac{49}{1765}}$$

Therefore, $\overset{w}{49} : \overset{w}{1765} :: \overset{d}{1} : \overset{d}{36\tfrac{1}{49}}$ Ans.

8. Five merchants, A, B, C, D, *and E, resolved* to trade in company, and agreed that A should advance $\frac{3}{7}$ of their capital, B $\frac{5}{21}$, C $\frac{1}{9}$, D $\frac{2}{15}$, and E the rest. By an unforeseen accident, C was prevented from joining the company; but A, B, D, and E advanced their respective shares according to agreement, and in one year, the profits of their trade amounted to $100000; how must the sum be divided among A, B, D, and E?

$\frac{3}{7}+\frac{5}{21}+\frac{1}{9}+\frac{2}{15}=\frac{9}{21}+\frac{5}{21}+\frac{5}{45}+\frac{6}{45}=\frac{2}{3}+\frac{11}{45}=\frac{30}{45}+\frac{11}{45}=\frac{41}{45}$ and $1-\frac{41}{45}=\frac{4}{45}=$ share advanced by E. The several shares of the gain will, therefore, be as $\frac{3}{7}$, $\frac{5}{21}$, $\frac{2}{15}$, and $\frac{4}{45}$, or as the least possible whole numbers, 135, 75, 42, 28. Hence then,

$$135+75+42+28=280 : \left\{\begin{array}{r} 135 :: \\ 75 :: \\ 42 :: \\ 28 :: \end{array}\right\} 100000 \left\{\begin{array}{l} : 48214\tfrac{2}{7} \text{ A's} \\ : 26785\tfrac{5}{7} \text{ B's} \\ : 15000 \text{ D's} \\ : 10000 \text{ E's} \end{array}\right.$$

9. Five political vagabonds, A, B, C, D, and E, are to be transported from the city of Philadelphia. The transportation of A, B, and C, will cost $101; of B, C, and D, $113; of C, D, and E, $116; of A, D, and E, $112; and of A, B, and E, $116; what will be the expense of transporting each?

Ans. A=27, B=43, C=31, D=39, and E= $46; for $27+43+31+39+46=186=\frac{558}{3}$, that is, the sum of the respective shares, is equal to one-third of the sum of the five combinations of every three of them.

10. The commanding officer of a certain post being sick, was informed by a physician, that if five handfuls of a certain herb could be immediately procured, the disease might be removed. The officer ordered five of his soldiers, A, B, C, D, and E, to go instantly in quest of the herb; and, as an inducement to return as quickly as possible, he promised to divide \$20 dollars amongst them, in a reciprocal proportion to the number of minutes in which each should present him with a handful of the herb. A returned in 5, B in 7, C in 9, D in 11, and E in 12 minutes. What share of the \$20 has each soldier to receive?

$$\text{Ans.}\begin{cases} 6.36\frac{6348}{8707} = \text{A's share.} \\ 4.54\frac{7022}{8707} = \text{B's.} \\ 3.53\frac{6429}{8707} = \text{C's.} \\ 2.89\frac{3677}{8707} = \text{D's.} \\ 2.65\frac{2645}{8707} = \text{E's.} \end{cases}$$

Proof \$20.00

11. At what times of the day do the hour and minute hands of a watch form a continued straight line?

Ans. At 6, 7 $5\frac{5}{12}$, 8 $10\frac{10}{11}$, 9 $16\frac{4}{11}$, 10 $21\frac{9}{11}$, 11 $27\frac{3}{11}$, 0 $32\frac{2}{11}$, 2 $43\frac{7}{11}$, 3 $49\frac{1}{11}$, and 4 $54\frac{6}{11}$.

(each time printed with *h* over the hours and *m* over the minutes)

12. It is required to divide 9 into two such parts, that when those parts divide each other, (the lesser that of the greater,) the quotient may be 99.

Solution. $99 + 1 = 100$, and $9 \div 100 = \frac{9}{100} =$ the lesser, and $9 - \frac{9}{100} = 8.\frac{91}{100}$, the greater; for $8.91 \div .09 = 99.$, or $\frac{9}{100})8.\frac{91}{100}(\frac{891}{9} = 99.$

Rule, for all such questions. Add to the given quotient 1, divide the given number by the quotient thus increased, and this quotient is the lesser number required. Again; subtract the lesser from their sum, and the difference will be the greater.

13. A person dying, left his wife with child; and by his will, ordered, that if she went with a son, $\frac{2}{3}$ of the estate should belong to him, and the remainder to his mother; and if she had a daughter, he appointed the mother $\frac{2}{3}$ and the daughter $\frac{1}{3}$; but it happened that she was delivered of both a son and a daughter, by which she lost in equity £2000 more than if it had only been a girl. What would have been her dowry, if she had had only a son?

It is evident from the nature of the question, that the estate should be so divided, that each of the three have shares proportional to those mentioned in the will,

That is, if the daughter has 1 share,
The mother must have 2
And of course the son 4

$$\overline{7}$$

Here the estate is divided into seven parts, of which the mother should have had $\frac{2}{3}$; that is, $\frac{2}{3}$ of $7=4\frac{2}{3}$, had she had only a daughter; and but $\frac{1}{3}$, that is, $\frac{1}{3}$ of $7,=2\frac{1}{3}$, had she had a son only; and $4\frac{2}{3}-2=2\frac{2}{3}$, her loss more than if she had only had a daughter.

Therefore, $2\frac{2}{3} : 2\frac{1}{3} :: 2000 : 1750$ Ans.

14. In what time will any sum, (as $100) double and treble itself, at 6 per cent compound interest?

$$1.\quad \frac{\text{Log. } 100\times 2-\text{log. } 100}{\text{Log. } 1.06}=11.895644+\text{ years.}$$

$$2.\quad \frac{\text{Log. } 100\times 3-\text{log. } 100}{\text{Log. } 1.06}=18.854811+\text{ years.}$$

15. A company of soldiers spent at a reckoning £3.15, and it so happened, that each man paid as many farthings as the number of persons in company. Required, the number of soldiers.

£3.15 = 3600*qrs.* and $\sqrt{3600}=60$. Ans.

16. Place the nine digits together, so as to make exactly 100.

17. How should I undertake to find two numbers, such, that their difference may be exactly 12, and their product exactly 6?

Half difference $=6$, and $\sqrt{6\times6+6}+6=$ greater, and $\sqrt{6\times6+6}-6$ the lesser; for $\sqrt{42}+6\times\sqrt{42}-6=42-36=6$.

Rule. To the given product add the square of half the given difference, the square root of this added to half the given difference is the greater, and if from the square root of the same, half the given difference be subtracted, it will be the lesser.

18. What number should I take, so that if I add 4 to it, the sum will be equal to 4 divided by the required number? Ans. $2\sqrt{2}-2$.

Rule. To the given number, add the square of its half; the square root of this sum, minus half the given number, will be the number required.

19. What two numbers shall I take, so that if three times the greater be added to five times the lesser, the sum will be equal to the greater divided by the lesser? Ans. $\frac{19}{62}$ and $\frac{361}{62}$.

20. In what order should 9 trees be planted, so that there will be 9 rows, and 3 trees to each row?

21. How should 7 trees be planted, so that the seventh tree may stand equally distant from each of the rest, each of the six being as distant from each other as from the seventh?

22. How should five trees be planted, so that the fifth may be equally distant from each of the rest, when each of the four are as distant from each other as from the fifth?

23. How should a gardener plant 19 apple trees, so that he may have 9 rows, and 5 trees in each row?

24. What three numbers should I take, that I may have 34 every time, when I add to the first half the sum of the remaining two, to the second, a third of the remaining two, and to the third, a fourth of the remaining two? Ans. 10, 22, and 26.

25. Suppose a mixture of gold, brass, iron, and tin, to weigh 60*lb.*, and the weight of gold and brass together 40*lb.*, the gold and tin 45*lb.*, and the gold and iron, 36*lb.*; how much was the weight of each?

Ans. Gold 30½*lb.*, brass 9½*lb.*, iron 5½*lb.*,
and tin 14½*lb.*

26. If 2 be 3, and 3 be 5, and 6½ be 11, what is the half of 26, and the fourth of 27?

Ans. 55, and $28\frac{29}{52}$.

27. If a stone of 40*lb.* weight was to *break* into four pieces, such that by help of them alone, a person might weigh 1, 2, 3, &c. *lbs.* to 40, what should the weight of each piece be?

Ans. 1, 3, 9, 27*lbs.* respectively.

28. The ages of A, B, and C are as follow; the age of A, together with half the sum of the other two, is 73; the age of B, together with one-third the sum of the other two, is 54; and the age of C, together with one-fourth of the sum of the other two, is 38. Their respective ages are required.

Ans. A's 48, B's 32, C's 18 years.

29. A landed man two daughters had,
And both were very fair;
To each he gave a piece of land,
One round the other square.
At twenty shillings an acre just
Each piece its value had;
The shillings that did compass each
For it exactly paid.
If 'cross a shilling be an inch,
As it is very near,
Which was the greater fortune, she
Which had the round, or square?

Ans. Area of square=250905.6 acres.
Area of circle=197060.63125 acres.
And of course each equal to as many pounds; which shew that she who had the square was much the better fortune. This question may be done by Position or by Algebra.

30. If a roll of butter weighs in one scale $2\frac{1}{4}$*lb.*, and being changed into the other, weighs 4*lb.*; what is the true weight of the butter?

Ans. 3*lb.*; for $\sqrt{2.25 \times 4} = 3$; which is a true rule for all such cases, viz. the square root of the product of the respective weights.

31. I have seen a pack of cards, with 32 numbers on each, so contrived, that the numbers in their natural order, from 1 to 63, were distributed amongst the cards in such a manner that the same numbers are to be found on 1, 2, 3, 4, 5, and 6, of them, and others but on one or two of the cards. By help of these cards, one can tell another whatever number he may chance to think upon, if it does not exceed the highest, provided he be informed of the cards on which this number may be found. It is required to construct these cards, and explain how one may guess to such certainty.

On the following page you see, that 1, 2, 4, 8, 16, and 32 are in the 1st horizontal row, and no where else; but $1+2=3$, shews that 3 is to be found in the first and second columns because it requires 1 and 2 to make 3. If it requires three or more of the top numbers to make one of the numbers in the table, as $1+2+4+8=15$, the number 15 is found in all the columns in which 1, 2, &c. are found at top; and so it is with all the other numbers, which are generated in the same way. The horizontal rank of numbers, 1, 2, 4, &c. are in geometrical ratio, and the perpendicular rank on the left in arithmetical ratio; both together, the abscisae and ordinates of the logarithmic curve. The rank of geometricals at the head, viz. 1, 2, 4, &c. afford such combinations as make up all the other numbers. In whatever columns any number stands, the sum of the geometricals which stand over such number, is equal to it; and the reason why one can guess any number thought on, is, the sum of the numbers which stand over it, in the first horizontal row, makes it, and no other, exactly.

1	2	4	8	16	32
3	3	5	9	17	33
5	6	6	10	18	34
7	7	7	11	19	35
9	10	12	12	20	36
11	11	13	13	21	37
13	14	14	14	22	38
15	15	15	15	23	39
17	18	20	24	24	40
19	19	21	25	25	41
21	22	22	26	26	42
23	23	23	27	27	43
25	26	28	28	28	44
27	27	29	29	29	45
29	30	30	30	30	46
31	31	31	31	31	47
33	34	36	40	48	48
35	35	37	41	49	49
37	38	38	42	50	50
39	39	39	43	51	51
41	42	44	44	52	52
43	43	45	45	53	53
45	46	46	46	54	54
47	47	47	47	55	55
49	50	52	56	56	56
51	51	53	57	57	57
53	54	54	58	58	58
55	55	55	59	59	59
57	58	60	60	60	60
59	59	61	61	61	61
61	62	62	62	62	62
63	63	63	73	63	63

EXAMPLE.

Suppose I should fix upon the number 50, and ask you to tell which I thought on; you would ask me in which of the columns, and how many, does it stand in? I would say, in the second and two last columns. You would then put together $2+16+32=50$, and tell me to a certainty; because 50 stands no where else but in the second and two last columns. Whatever number you guess upon, if only in the first, second, third, fourth, fifth, or sixth columns, it must be 1, 2, 4, 8, 16, or 32; because each of them are found but once; but where it takes two or more of the figures in the horizontal row of geometricals, to make the one you think on, the number is to be found in all the columns, exactly under the geometricals which compose it, and no where else. Hence it is easy both to make and understand them; and even to extend the cards to much higher numbers. If another column was added, its head number would be 64; from which a table could be generated, that would give all the numbers from 1 to 127; because $63+64=127$.

32. Divide 40 into four such parts, that if to the first you add 4, from the second subtract 4, multiply the third by 4, and divide the fourth by 4, the sum, difference, product, and quotient will be all equal.
Ans. $\frac{4}{5}$, $\frac{44}{5}$, $\frac{6}{5}$, and $\frac{96}{5}$.

33. Into what three whole parts can 117 be divided, so that if each of them be squared, their squares will be in arithmetical ratio? Ans. 9, 45, 63.

34. Divide 50 into four such parts, that when each of those parts are cubed, the sum of their cubes may be a cube.